10 Universal Gravitation

Things such as leaves, rain, and satellites fall because of gravity. Gravity is what holds tea in a cup and what makes bubbles rise. It made the earth round, and it builds up the pressures which kindle every star that shines. These are things that gravity does. But what *is* gravity? We do not know what gravity is—at least not in the sense that we know what sound, heat, and light are. We have names for things we understand, and we also have names for things we do not understand. Gravity is the name we give to the force of attraction between objects, even though we do not thoroughly understand it. Nevertheless, we understand how gravity affects things such as projectiles, satellites, and the solar system of planets. We also understand that gravity extends throughout the universe; it accounts for such things as the shapes of galaxies. This chapter covers the basic behavior of gravity. In the next chapter we shall investigate more of its consequences.

10.1 The Falling Apple

The idea that gravity extends throughout the universe is credited to Isaac Newton. According to popular legend, the idea occurred to Newton while he was sitting underneath an apple tree on his mother's farm pondering the forces of nature. Newton understood the concept of inertia developed earlier by Galileo; he knew that without an outside force, moving objects continue to move at constant speed in a straight line. He knew that if an object undergoes a change in speed or direction, then a force is responsible.

A falling apple triggered what was to become one of the most far-reaching generalizations of the human mind. Newton saw the apple fall, or maybe even felt it fall on his head—the story about this is not clear. Looking up through the apple tree branches toward the origin of the apple, did Newton notice the moon? Newton had been giving a lot of thought to the fact that the moon does not follow a straight line path, but instead circles about the earth. Now, circular motion is accelerated motion, which requires a force. But what was this force? Newton had the *insight* to see that the force that pulls between the earth and moon is the same force that pulls between apples and everything else in our universe. This force is the force of gravity.

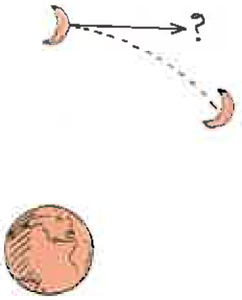

Fig. 10-1 If the moon did not fall, it would follow the straight-line path. Because of its attraction to the earth, it falls along a curved path.

10.2 The Falling Moon

Newton developed this idea further. He compared the falling apple with the falling moon. Does the moon fall? Yes, it does. Newton realized that if the moon did not fall, it would move off in a straight line and leave the earth. His idea was that the moon must be falling *around* the earth. Thus the moon falls in the sense that it *falls beneath the straight line it would follow if no force acted on it*. He hypothesized that the moon was simply a projectile circling the earth under the attraction of gravity.

This idea is illustrated in an original drawing by Newton, shown in Figure 10-2. He compared motion of the moon with a cannonball fired from the top of a high mountain. He imagined that the mountaintop was above the earth's atmosphere, so that air resistance would not impede the motion of the cannonball. If a cannonball were fired with a small horizontal speed, it would follow a parabolic path and soon hit the earth below. If it were fired faster, its path would be less curved and it would hit the earth farther away. If the cannonball were fired fast enough, Newton reasoned, the parabolic path would become a circle and the cannonball would circle indefinitely. It would be in orbit.

Both the orbiting cannonball and the moon have a component of velocity parallel to the earth's surface. This sideways velocity, or **tangential velocity**, is sufficient to insure motion *around* the earth rather than *into* it. If there is no resistance to reduce its speed, the moon "falls" around and around the earth indefinitely.

Newton's idea seemed correct. But for the idea to advance from hypothesis to the status of a scientific theory, it would have to be tested. Newton's test was to see if the moon's "fall" beneath its otherwise straight-line path was in correct proportion to the

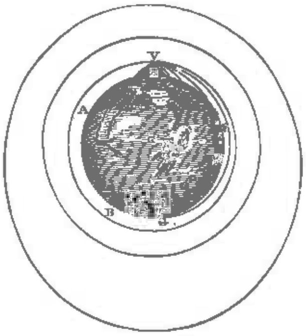

Fig. 10-2 The original drawing by Isaac Newton showing how a projectile fired fast enough would fall around the earth and become an earth satellite. In the same way, the moon falls around the earth and is an earth satellite.

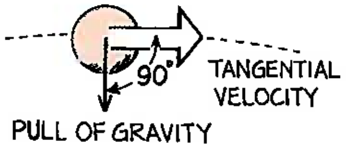

Fig. 10-3 Tangential velocity is the "sideways" velocity. That is, it is the component of velocity that is perpendicular to the pull of gravity.

fall of an apple or any object at the earth's surface. He reasoned that the mass of the moon should not affect how it falls, just as mass has no effect on the acceleration of freely-falling objects on earth. How far the moon falls, and how far an apple at the earth's surface falls, should relate only to their respective *distances* from the earth's center. If the distance of fall for the moon and the apple are in correct proportion, then the hypothesis that earth gravity reaches to the moon must be taken seriously.

The moon was already known to be 60 times farther from the center of the earth than an apple at the earth's surface. The apple will fall nearly 5 m in its first second of fall—or more accurately, 4.9 m. Newton reasoned that gravitational attraction to the earth must be "diluted" by distance. Does this mean the force of earth gravity would reduce to $\frac{1}{60}$ at the moon's distance? No, it is much less than this. As we shall soon see, the influence of gravity should be diluted $\frac{1}{60}$ of $\frac{1}{60}$, or $1/(60)^2$. So in one second the moon should fall $1/(60)^2$ of 4.9 m, which is 1.4 millimeters.*

Using geometry, Newton calculated how far the circle of the moon's orbit lies below the straight-line distance the moon other-

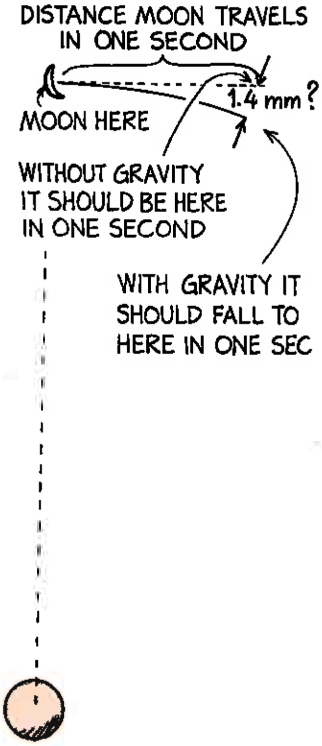

Fig. 10-4 If the force that pulls apples off trees also pulls the moon into orbit, the circle of the moon's orbit should fall 1.4 mm below a point along the straight line where the moon would otherwise be one second later.

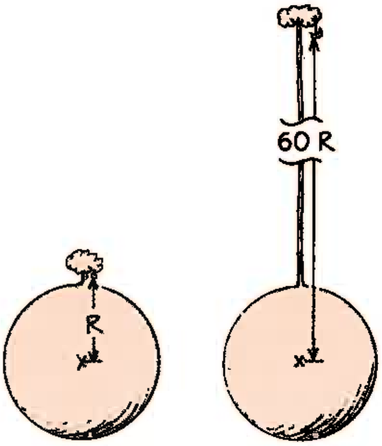

Fig. 10-5 An apple falls 4.9 m during its first second of fall when it is near the earth's surface. Newton asked how far the moon would fall in the same time if it were 60 times farther from the center of the earth. His answer was (in today's units) 1.4 mm.

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[*] Or working backwards, $(0.0014\text{ m}) \times (60)^2 = 4.9\text{ m}$.

wise would travel in one second (Figure 10-4). The distance should have been 1.4 mm. But he was disappointed to end up with a large discrepancy. Recognizing that a hypothesis, however elegant, is not valid if it cannot be backed up with tests, he placed his papers in a drawer, where they remained for nearly 20 years. During this period he laid the foundation and developed the field of geometrical optics for which he first became famous.

It turns out that Newton used an incorrect figure in his calculations. When he finally returned to the moon problem at the prodding of his astronomer friend Edmund Halley (of Halley Comet fame) and used a corrected figure, he obtained excellent agreement. Only then did he publish what is one of the greatest achievements of the human mind—the law of universal gravitation.* Newton generalized his moon finding to all objects, and stated that all objects in the universe attract each other.

10.3 The Falling Earth

Newton's theory of gravitation confirmed the Copernican theory of the solar system. It was now clear that the earth and planets orbit the sun in the same way that the moon orbits the earth. The mighty sun and planets simply pull each other so that the planets continually "fall" around the sun in closed paths. Why don't the planets crash into the sun? They don't because of their tangential velocities. What would happen if their tangential velocities were reduced to zero? The answer is simple enough: their motion would be straight toward the sun and they would indeed crash into it. Any objects in the solar system with insufficient tangential velocities have long ago crashed into the sun. What remains is the harmony we observe.

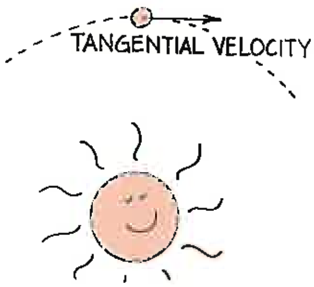

Fig. 10-6 The tangential velocity of the earth about the sun allows it to fall around the sun rather than directly into it. If this tangential velocity were reduced to zero, what would be the fate of the earth?

▸ **Question**

Since the moon is gravitationally attracted to the earth, why does it not simply crash into the earth?

▸ **Answer**

The moon would crash into the earth if its tangential velocity were reduced to zero, but because of its tangential velocity, the moon falls around the earth rather than into it. We will return to this idea in more detail in the next chapter.

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* This is a dramatic example of the painstaking effort and crosschecking that go into the formulation of a scientific theory. Compare this with the lack of "doing one's homework," the hasty judgements, and the absence of cross-check examinations that so often characterize the pronouncement of less-than-scientific theories.

10.4 The Law of Universal Gravitation

Newton did not discover gravity. What Newton discovered was that gravity was universal. Everything pulls on everything else in a beautifully simple way that involves only mass and distance. Newton's **law of universal gravitation** states that every object attracts every other object with a force that for any two objects is directly proportional to the mass of each object. The greater the masses, the greater the force of attraction between them. Newton deduced that the force decreases as the square of the distance between the centers of mass of the objects. The farther away the objects are from each other, the less the force of attraction between them.

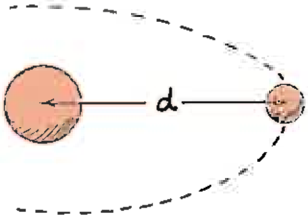

Fig. 10-7 The force of gravity between objects depends on the distance between their centers of mass.

The law can be expressed symbolically as

$$F \sim \frac{m_1 m_2}{d^2}$$

where m_1 is the mass of one object, m_2 is the mass of the other, and d is the distance between their centers of mass. The greater the masses m_1 and m_2, the greater the force of attraction between them. The greater the distance d between the objects, the weaker the force of attraction.

We can express the symbolic statement as an exact equation if we introduce the quantity G, the **universal constant of gravitation**. G is a conversion factor needed to change the units of mass and distance on the right side of the equation to the units of force on the left (just as we use conversion factors to change from miles to kilometers, for example). Then we can express the law of universal gravitation as the exact equation

$$F = G\frac{m_1 m_2}{d^2}$$

In words, the force of gravity between two objects is found by multiplying their masses, dividing by the square of the distance between them, then multiplying this result by the constant G. The value of G was determined by actual measurement long after Newton's time. It is

$$G = 0.0000000000667\ \frac{\mathrm{N{\cdot}m^2}}{\mathrm{kg^2}}$$

or, in scientific notation,*

$$G = 6.67 \times 10^{-11}\ \mathrm{N{\cdot}m^2/kg^2}$$

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* Scientific notation is discussed in Appendix A at the end of this book.

The value of G tells us that the force of gravity is a very weak force. It is the weakest of the four fundamental forces found in nature. (The other three are the electromagnetic force and two kinds of nuclear forces.) We sense gravitation only when masses like that of the earth are involved. The force of attraction between a pair of 1-kg masses with their centers of gravity 1 m apart is only 6.67×10^{-11} N, too tiny for ordinary measurement. The force of attraction between you and the earth, however, can be measured. It is called your *weight*.

Your weight is the gravitational attraction between your mass and the mass of the earth. If you gain mass, you gain weight. Or if the earth somehow gained mass, you'd also gain weight. Your weight also depends on your distance from the center of the earth. At the top of a mountain your mass is no different than it is anywhere else, but your weight is slightly less than at ground level. This is because your distance from the center of the earth is greater.

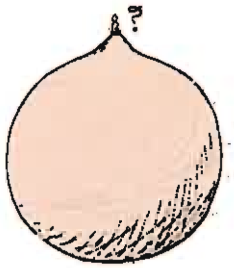

Fig. 10-8 Your weight is less at the top of a mountain because you are farther from the center of the earth.

▶ Question

If there is an attractive force between all objects, why do we not feel ourselves gravitating toward massive buildings in our vicinity?

10.5 Gravity and Distance: The Inverse-Square Law

We can understand how gravity is reduced with distance by considering an imaginary "butter gun" used in a busy restaurant for buttering toast (Figure 10-9). Imagine melted butter sprayed through a square opening in a wall. The opening is exactly the size of one piece of square toast. And imagine that a spurt from the gun deposits an even layer of butter 1 mm thick. Consider the consequences of holding the toast twice as far from the butter gun. You can see in Figure 10-9 that the butter would spread out

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▶ Answer

We *are* gravitationally attracted to massive buildings and everything else in the universe. The 1933 Nobel prize-winning physicist Paul A.M. Dirac put it this way: "Pick a flower on earth and you move the farthest star!" How *much* you are influenced by buildings or how much interaction there is between flowers, is another story. The forces between you and buildings are relatively small because the masses are small compared to the mass of the earth. The forces due to the stars are small because of their great distances. These tiny forces escape our notice when they are overwhelmed by the overpowering attraction to the earth.

for twice the distance and would cover twice as much toast vertically and twice as much toast horizontally. A little thought will show that the butter would now spread out to cover four pieces of toast. How thick will the butter be on each piece of toast? Since it has been diluted to cover four times as much area, its thickness will be one-quarter as much, or 0.25 mm.

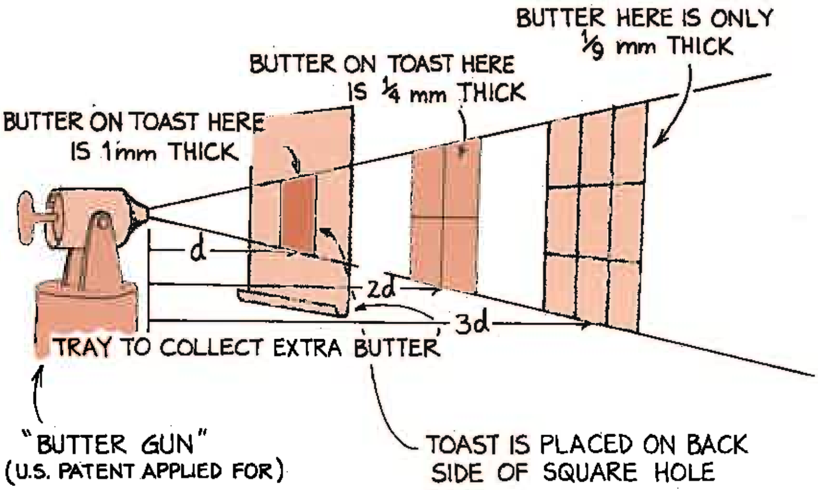

Fig. 10-9 The inverse-square law. Butter spray travels outward from the nozzle of the butter gun in straight lines. Like gravity, the "strength" of the spray obeys the inverse-square law.

Note what has happened. When the butter gets twice as far from the gun, it is only $\frac{1}{4}$ as thick. More thought will show that if it gets 3 times as far, it will spread out to cover 3 × 3, or 9, pieces of toast. How thick will the butter be then? Can you see it will be $\frac{1}{9}$ as thick? And can you see that $\frac{1}{9}$ is the inverse square of 3? (The inverse of 3 is simply $\frac{1}{3}$; the inverse *square* of 3 is $(\frac{1}{3})^2$, or $\frac{1}{9}$.) This law applies not only to the spreading of butter from a butter gun, and the weakening of gravity with distance, but to all cases where the effect from a localized source spreads evenly throughout the surrounding space. More examples are light, radiation, and sound.

The greater the distance from the earth's center, the less an object will weigh (Figure 10-10). If your little sister weighs 300 N at sea level, she will weigh only 299 N atop Mt. Everest. But no matter how great the distance, the earth's gravity does not drop to zero. Even if you were transported to the far reaches of the universe, the gravitational influence of the earth would be with you. It may be overwhelmed by the gravitational influences of nearer and/or more massive objects, but it is there. The gravitational influence of every object, however small or far, is exerted through all space. Isn't that amazing?

▶ **Question**

Suppose that an apple at the top of a tree is pulled by earth gravity with a force of 1 N. If the tree were twice as tall, would the force of gravity on the apple be only ¼ as strong? Explain your answer.

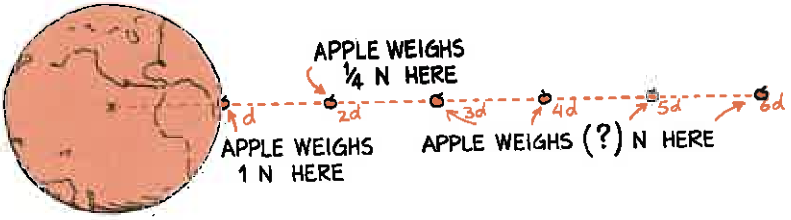

Fig. 10-10 An apple that weighs 1 N at the earth's surface weighs only 0.25 N when located twice as far from the earth's center because the pull of gravity is only ¼ as strong. When it is 3 times as far, it weighs only $\frac{1}{9}$ as much, or 0.11 N. What would it weigh at 4 times the distance? Five times?

10.6 Universal Gravitation

We all know that the earth is round. But why is the earth round? It is round because of gravitation. Since everything attracts everything else, the earth has attracted itself together before it became solid. Any "corners" of the earth have been pulled in so that the earth is a giant sphere. The sun, the moon, and the earth are all fairly spherical because they have to be (rotational effects make them somewhat wider at their equators).

If everything pulls on everything else, then the planets must pull on each other. The net force that controls Jupiter, for example, is not just from its interaction with the sun, but with the planets also. Their effect is small compared to the pull of the more massive sun, but it still shows. When the planet Saturn is near Jupiter, for example, its pull disturbs the otherwise smooth path of Jupiter. Both planets deviate from their normal orbits. This deviation is called a **perturbation**.

▶ **Answer**

No, because the twice-as-tall apple tree is not twice as far from the *earth's center*. The taller tree would have to have a height equal to the radius of the earth (6370 km) before the weight of the apple reduces to ¼ N. Before its weight decreases by one percent, an apple or any object must be raised 32 km—nearly 4 times the height of Mt. Everest, the tallest mountain in the world. So as a practical matter we disregard the effects of everyday changes in elevation.

Up until the middle of the last century astronomers were puzzled by unexplained perturbations of the planet Uranus. Even when the influences of the other planets were taken into account, Uranus was behaving stangely. Either the law of gravitation was failing at this great distance from the sun, or some unknown influence such as another planet was perturbing Uranus.

The source of Uranus's perturbation was uncovered in 1845 and 1846 by two astronomers, John Adams in England, and Urbain Leverrier in France. With only pencil and paper and the application of Newton's law of gravitation, both astronomers independently arrived at the same conclusion: a disturbing body beyond the orbit of Uranus was the culprit. They sent letters to their local observatories with instructions to search a certain part of the sky. The request by Adams was delayed by misunderstandings at Greenwich, England, but Leverrier's request to the director of the Berlin observatory was heeded right away. The planet Neptune was discovered within a half hour.

Other perturbations of Uranus led to the prediction and discovery of the ninth planet, Pluto. It was discovered in 1930 at the Lowell observatory in Arizona. Pluto takes 248 years to make a single revolution about the sun, so it will not be seen in its discovered position again until the year 2178.

The perturbations of double stars and the shapes of distant galaxies are evidence that the law of gravitation extends beyond the solar system. Over still larger distances, gravitation dictates the fate of the entire universe.

Fig. 10-11 Gravitation dictates the shape of the spiral arms in a galaxy.

Current scientific speculation is that the universe originated in the explosion of a primordial fireball some 15 to 20 billion years ago. This is the "Big Bang" theory of the origin of the universe. All the matter of the universe was hurled outward from this event and continues in an outward expansion. We find ourselves in an expanding universe.

This expansion may go on indefinitely, or it may be overcome by the combined gravitation of all the galaxies and come to a stop. Like a stone thrown upward, whose departure from the ground comes to an end when it reaches the top of its trajectory, and which then begins its descent to the place of its origin, the universe may contract and fall back into a single unity. This would be the "Big Crunch." After that, the universe would presumably re-explode to produce a new universe. The same course of action might repeat itself and the process may well be cyclic. If this is true, we live in an oscillating universe.

We do not know whether the expansion of the universe is cyclic or indefinite, because we are uncertain about whether enough mass exists to halt the expansion. The period of oscillation is estimated to be somewhat less than 100 billion years. If the universe does oscillate, who can say how many times it has expanded and collapsed? We know of no way a civilization could leave a trace of ever having existed during a previous cycle, for all the matter in the universe would be reduced to bare subatomic particles during the collapse. All the laws of nature, such as the law of gravitation, would then have to be rediscovered by the higher evolving life forms. And then students of these laws would read about them, as you are doing now. Think about that.

Few theories have affected science and civilization as much as Newton's theory of gravity. The successes of Newton's ideas ushered in the Age of Reason or Century of Enlightenment. Newton had demonstrated that by observation and reason, people could uncover the workings of the physical universe. How profound that all the moons and planets and stars and galaxies have such a beautifully simple rule to govern them, namely

$$F = G\frac{m_1 m_2}{d^2}$$

The formulation of this simple rule is one of the major reasons for the success in science that followed, for it provided hope that other phenomena of the world might also be described by equally simple laws.

This hope nurtured the thinking of many scientists, artists, writers, and philosophers of the 1700s. One of these was the English philosopher John Locke, who argued that observation and reason, as demonstrated by Newton, should be our best judge and guide in all things. Locke urged that all of nature and even society should be searched to discover any "natural laws" that might exist. Using Newtonian physics as a model of reason, Locke and his followers modeled a system of government that found adherents in the 13 British colonies across the Atlantic. These ideas culminated in the Declaration of Independence and the Constitution of the United States of America.

10 Chapter Review

Concept Summary

The moon and other objects in orbit around the earth are actually falling toward the earth but have great enough tangential velocity to avoid hitting the earth.

According to Newton's law of universal gravitation, everything pulls on everything else with a force that depends upon the masses of the objects and the distances between their centers of mass.

- The greater the masses, the greater is the force.
- The greater the distance, the smaller is the force.

Important Terms

law of universal gravitation (10.4)
perturbation (10.6)
tangential velocity (10.2)
universal constant of gravitation (10.4)

Review Questions

1. Why did Newton think that a force must act on the moon? (10.1)
2. What was it that Newton discovered about the force that pulls apples to the ground and the force that holds the moon in orbit? (10.1)
3. If the moon falls, why doesn't it get closer to the earth? (10.2)
4. What is meant by *tangential* velocity? (10.2)
5. How did Newton check his hypothesis that there is an attractive force between the earth and moon? (10.2)
6. What is required before a hypothesis (an educated guess) advances to the status of a scientific theory (organized knowledge)?
7. Since the planets are pulled to the sun by gravitational attraction, why don't they simply crash into the sun? (10.3)
8. Newton did not *discover* gravity but, rather, something about gravity. What was his discovery? (10.4)
9. What does the very small value of the gravitational constant G tell us about the strength of gravitational forces? (10.4)
10. Exactly what are the two specific masses and the one specific distance that determine your weight? (10.4)
11. In what way is gravity reduced with distance from the earth? (10.5)
12. What would be the difference in your weight if you were five times farther from the center of the earth as you are now? Ten times? (10.5)
13. What makes the earth round? (10.6)
14. What causes planetary perturbations? (10.6)
15. Distinguish between the "Big Bang" and the "Big Crunch." (10.6)

Think and Explain

1. The moon "falls" 1.4 mm each second. Does this mean that it gets 1.4 mm closer to the earth each second? Would it get closer if its tangential velocity were reduced? Defend your answer.

2. Comparing a large boulder with a small stone, which is attracted to the earth with the greater force? Which undergoes the greater acceleration when falling without air resistance? Defend both answers.

3. If the gravitational forces of the sun on the planets suddenly disappeared, in what kind of paths would they move?

4. If the moon were twice as massive, would the attractive force between the earth and moon be twice as large? Between the moon and earth? Defend your answer.

5. a. If the gravitational force between two massive bodies were measured and divided by the product of their masses, and then multiplied by the square of the distance between their centers of mass, what number would result?
   b. Would this number differ for different masses at different distances? Defend your answer.

6. If the moon were twice as far away and remained in circular orbit about the earth, by what distance would it "fall" each second?

7. If you stood atop a ladder that was so tall that you were twice as far from the earth's center, how would your weight compare to its present value?

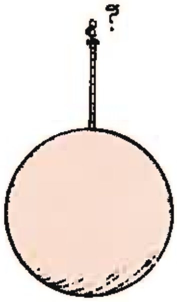

Fig. A

8. By what factor would your weight change if the earth were twice as big and had twice the mass?

9. Evidence indicates that the present expansion of the universe is slowing down. Is this consistent with, or contrary to, the law of gravity? Explain.

10. Some people dismiss the validity of scientific theories by saying they are "only" theories. The law of universal gravitation is a theory. Does this mean that scientists still doubt its validity? Explain.