12 Satellite Motion

If you drop a stone from a rest position to the ground below, it will fall in a straight-line path. If you move your hand horizontally as you drop the stone, it will follow a curved path to the ground. If you move your hand faster, the stone lands farther away and the curvature of the path is less pronounced. What would happen if the curvature of the path matched the curvature of the earth? The answer is simple enough: if air resistance can be neglected, you would have an earth satellite.

Fig. 12-1 The greater the stone's horizontal motion when released, the wider the arc of its curved path.

12.1 Earth Satellites

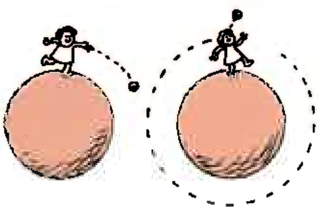


Fig. 12-2 If you toss the stone horizontally with the proper speed, its path will match the surface curvature of the asteroid.

An earth satellite is simply a projectile that falls *around* the earth rather than *into* it. This is easier to see if you imagine yourself on a "smaller earth," perhaps an asteroid (Figure 12-2). Because of the small size and low mass, you will not have to throw the stone very fast to make its curved path match the surface curvature of the asteroid. If you toss the stone just right, it will follow a circular orbit.

How fast would the stone have to be thrown horizontally in order to orbit the earth? The answer depends on the rate at which it falls and the degree to which the earth curves. Recall from Chapter 2 that a stone dropped from rest will accelerate 9.8 m/s² and fall a vertical distance of 4.9 m during the first second of fall. Also recall from Chapter 6 that the same is true of anything already moving as it starts to fall, that is, a projectile. Remember that in the first second a projectile will fall a vertical distance of 4.9 m from where it would have gone without gravity. (It may be helpful to refresh your memory and review Figure 6-15 on page 78.) So in the first second after a stone is thrown, it will be 4.9 m below the straight-line path it would have taken if there were no gravity.

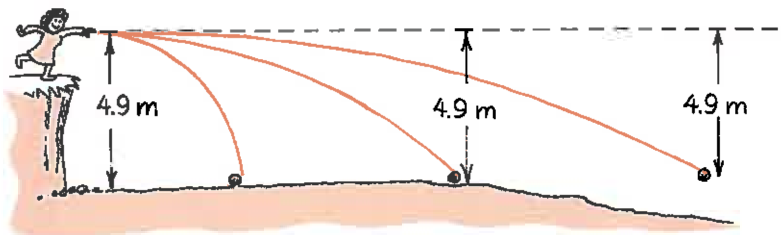


Fig. 12-3 Throw a stone at any speed and one second later it will have fallen 4.9 m below where it would have been without gravity.

A geometrical fact about the curvature of our earth is that its surface drops a vertical distance of 4.9 m for every 8000 m tangent to the surface (Figure 12-4). This means that if you were swimming in a calm ocean, you would be able to see only the top of a 4.9-m mast on a ship 8 km away.

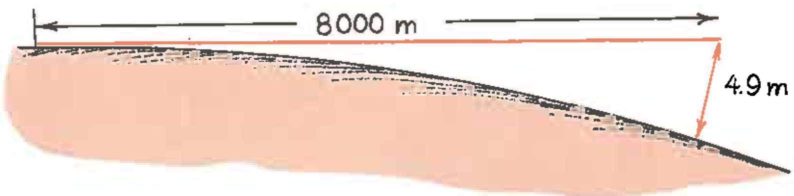
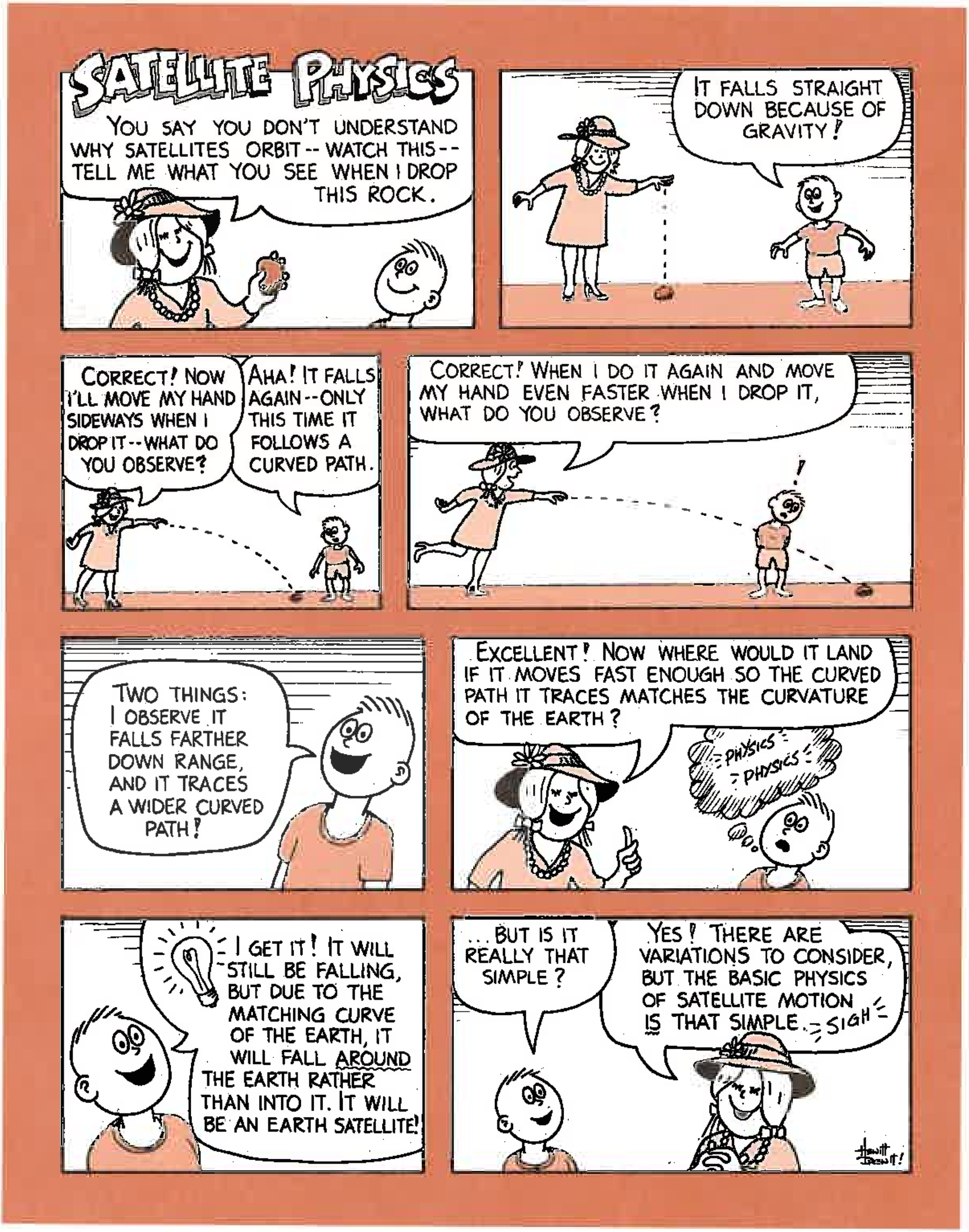


Fig. 12-4 The earth's curvature (not to scale).

If a stone could be thrown fast enough to travel a horizontal distance of 8 km during the time (1 s) it takes to fall 4.9 m, then can you see it will follow the curvature of the earth? Isn't this speed simply 8 km/s? So we see that the orbital speed for close orbit about the earth is 8 km/s. If this doesn't seem to be very fast, convert it to kilometers per hour; you'll see it is an impressive 29 000 km/h (or 18 000 mi/h). At this speed, atmospheric friction would burn a stone to a crisp. That is why a satellite must stay about 150 km or so above the earth's surface to keep from burning up like a "falling star" against the friction of the atmosphere.


SATELLITE PHYSICS
YOU SAY YOU DON'T UNDERSTAND WHY SATELLITES ORBIT -- WATCH THIS -- TELL ME WHAT YOU SEE WHEN I DROP THIS ROCK.
IT FALLS STRAIGHT DOWN BECAUSE OF GRAVITY!
CORRECT! NOW I'LL MOVE MY HAND SIDEWAYS WHEN I DROP IT -- WHAT DO YOU OBSERVE?
AHA! IT FALLS AGAIN -- ONLY THIS TIME IT FOLLOWS A CURVED PATH.
CORRECT! WHEN I DO IT AGAIN AND MOVE MY HAND EVEN FASTER WHEN I DROP IT, WHAT DO YOU OBSERVE?
!
TWO THINGS: I OBSERVE IT FALLS FARTHER DOWN RANGE, AND IT TRACES A WIDER CURVED PATH!
EXCELLENT! NOW WHERE WOULD IT LAND IF IT MOVES FAST ENOUGH SO THE CURVED PATH IT TRACES MATCHES THE CURVATURE OF THE EARTH?
PHYSICS PHYSICS
I GET IT! IT WILL STILL BE FALLING, BUT DUE TO THE MATCHING CURVE OF THE EARTH, IT WILL FALL AROUND THE EARTH RATHER THAN INTO IT. IT WILL BE AN EARTH SATELLITE!
... BUT IS IT REALLY THAT SIMPLE?
YES! THERE ARE VARIATIONS TO CONSIDER, BUT THE BASIC PHYSICS OF SATELLITE MOTION IS THAT SIMPLE. SIGH

12.2 Circular Orbits

Interestingly enough, in circular orbit the speed of a satellite is not changed by gravity. We can understand this by comparing a satellite in circular orbit with a bowling ball rolling along a bowling alley. Why doesn't the gravity that acts on the bowling ball change its speed? The answer is that gravity is pulling neither a bit forward nor a bit backward: gravity pulls straight downward. The bowling ball has no component of gravitational force along the direction of the alley.

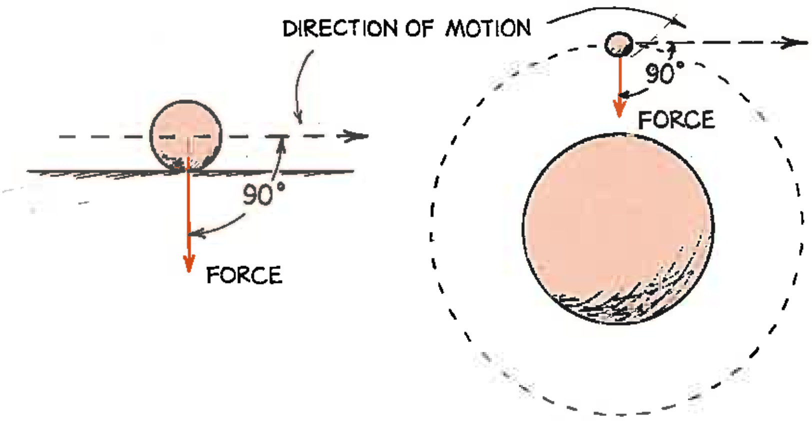


Fig. 12-5 (Left) The force of gravity on the bowling ball does not affect its speed because there is no component of gravitational force horizontally.(Right) The same is true for the satellite in circular orbit. In both cases, the force of gravity is at right angles to the direction of motion.

The same is true for a satellite in circular orbit. In circular orbit a satellite is always moving perpendicularly to the force of gravity. It does not move in the direction of gravity, which would increase its speed, nor does it move in a direction against gravity, which would decrease its speed. Instead, the satellite exactly "criss-crosses" gravity, with the result that no change in speed occurs—only a change in direction. A satellite in circular orbit around the earth moves parallel to the surface of the earth at constant speed.

For a satellite close to the earth, the time for a complete orbit about the earth, called the **period**, is about 90 minutes. For higher altitudes, the orbital speed is less and the period is longer. Communications satellites located in orbit 5.5 earth radii above the surface of the earth, for example, have a period of 24 hours. Their period of satellite rotation matches the period of daily earth rotation. Their orbit is around the equator, so they stay above the

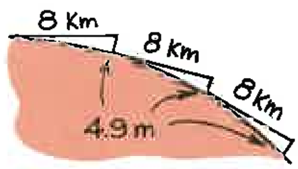


Fig. 12-6 A satellite in circular orbit close to the earth moves tangentially at 8 km/s. During each second, it falls 4.9 m beneath each successive 8-km tangent.

same point on the equator. The moon is even farther away and has a period of 27.3 days. The higher the orbit of a satellite, the less its speed and the longer its period.*

▶ **Questions**

1. There are usually alternate explanations for things. Is the following explanation valid? Satellites remain in orbit instead of falling to the earth because they are beyond the main pull of earth's gravity.
2. Satellites in close circular orbit fall about 4.9 m during each second of orbit. How can this be if the satellite does not get closer to the earth?

Recall from the last chapter that satellite motion was understood by Isaac Newton. He stated that at a certain speed a cannonball would circle the earth and coast indefinitely, provided air resistance could be neglected. Newton calculated the required speed to be equivalent to 8 km/s. Since such a cannonball speed was clearly impossible, he did not foresee people launching satellites. Newton did not consider multi-stage rockets.

12.3 Elliptical Orbits

If a projectile just above the drag of the atmosphere is given a horizontal speed somewhat greater than 8 km/s, it will overshoot a circular path and trace an oval-shaped path, an **ellipse**.

▶ **Answers**

1. No, no, a thousand times no! If any moving object were beyond the pull of gravity, it would move in a straight line and would not curve around the earth. Satellites remain in orbit because they *are* being pulled by gravity, not because they are beyond it.
2. In each second, the satellite falls about 4.9 m below the straight-line tangent it would have taken if there were no gravity. The earth's surface curves 4.9 m beneath a straight-line tangent that is 8 km long. Since the satellite moves at 8 km/s, it "falls" at the same rate that the earth "curves."

* If you continue with your study of physics and take a follow-up course, you'll learn that the speed v of a satellite in circular orbit is given by $v = \sqrt{GM/d}$ and the period T of satellite motion is given by $T = 2\pi\sqrt{d^3/GM}$, where G is the universal gravitational constant (see Chapter 10), M is the mass of the earth (or whatever body the satellite orbits), and d is the altitude of the satellite measured from the center of the earth or parent body.

An ellipse is a specific curve: the closed path taken by a point that moves in such a way that the sum of its distances from two fixed points (called **foci**) is constant. In the case of a satellite orbiting a planet, the center of the planet is at one focus (singular of *foci*); the other focus is empty. An ellipse can be easily constructed by using a pair of tacks, one at each focus, a loop of string, and a pencil, as shown in Figure 12-7. The closer the foci are to each other, the closer the ellipse is to a circle. When both foci are together, the ellipse *is* a circle. A circle is actually a special case of an ellipse with both foci at the center.

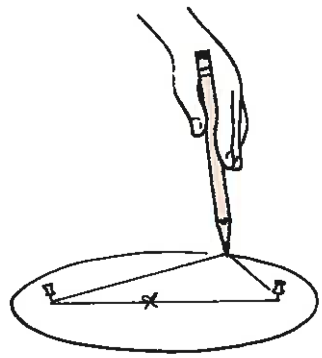


Fig. 12-7 A simple method of constructing an ellipse.

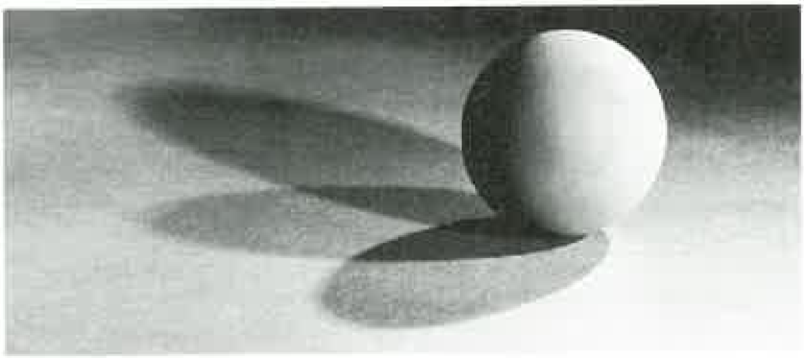


Fig. 12-8 The shadows of the ball are all ellipses with one focus where the ball touches the table.

Whereas the speed of a satellite is constant in a circular orbit, speed varies in an elliptical orbit. When the initial speed is greater than 8 km/s, the satellite overshoots a circular path and moves away from the earth, against the force of gravity. It therefore loses speed. Like a rock thrown into the air, it slows to a point where it no longer recedes, and it begins to fall back toward the earth. The speed it lost in receding is regained as it falls back toward the earth, and it finally crosses its original path with the same speed it had initially (Figure 12-9). The procedure repeats over and over, and an ellipse is traced each cycle.

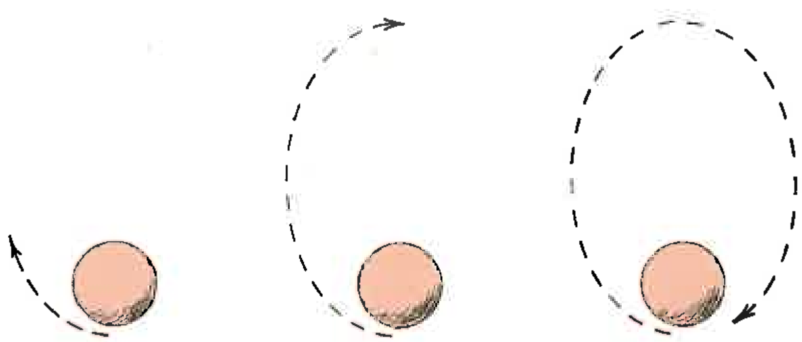
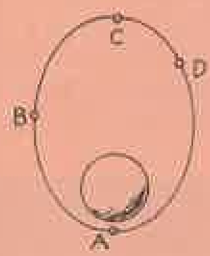


Fig. 12-9 Elliptical orbit. When the satellite exceeds 8 km/s, it overshoots a circle (left) and travels away from the earth against gravity. At its maximum separation (center) it starts to come back toward the earth. The speed it lost in going away is gained in returning, and (right) the cycle repeats itself.

▶ **Question**

The orbital path of a satellite is shown in the sketch. In which of the marked positions *A* through *D* does the satellite have the greatest speed? Lowest speed?



12.4 Energy Conservation and Satellite Motion

Recall from Chapter 8 that an object in motion possesses kinetic energy (KE) by virtue of its motion. An object above the earth's surface possesses potential energy (PE) by virtue of its position. Everywhere in its orbit, a satellite has both KE and PE with respect to the body it orbits. The sum of the KE and PE will be a constant all through the orbit. The simplest case occurs for a satellite in circular orbit.

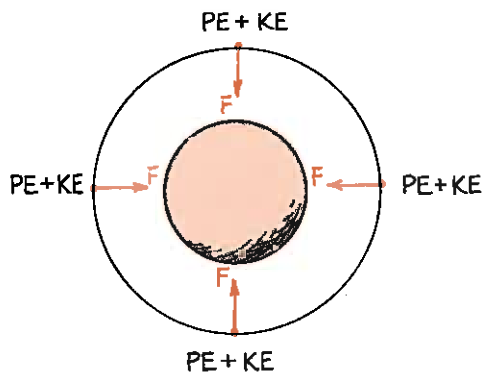


Fig. 12-10 The force of gravity on the satellite is always toward the center of the body it orbits. For a satellite in circular orbit, no component of force acts along the direction of motion. The speed, and thus the KE, cannot change.

▶ **Answer**

The satellite has its greatest speed as it whips around *A*. It has its lowest speed at position *C*. Beyond *C* it gains speed as it falls back to *A* to repeat its cycle.

In circular orbit the distance between the body's center and the satellite does not change. This means that the PE of the satellite is the same everywhere in orbit. Then, by the conservation of energy, the KE must also be constant. In other words, the speed is constant in any circular orbit.

In elliptical orbit the situation is different. Both speed and distance vary. The PE is greatest when the satellite is farthest away (at the **apogee**) and least when the satellite is closest (at the **perigee**). Correspondingly, the KE will be least when the PE is most; and the KE will be most when the PE is least. At every point in the orbit, the sum of the KE and PE is the same.

At all points along the orbit there is a component of gravitational force in the direction of motion of the satellite (zero at the apogee and perigee). This component changes the speed of the satellite. Or we can say: (this component of force) × (distance moved) = change in KE. Either way we look at it, when the satellite gains altitude and moves against this component, its speed and KE decrease. The decrease continues to the apogee. Once past the apogee, the satellite moves in the same direction as the component, and the speed and KE increase. The increase continues until the satellite whips past the perigee and repeats the cycle.

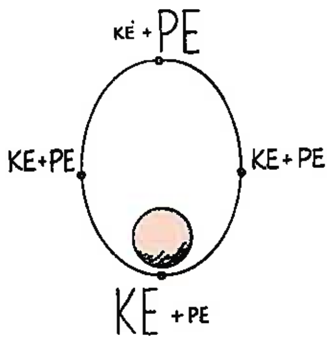


Fig. 12-11 The sum of KE and PE for a satellite is a constant at all points along an elliptical orbit.

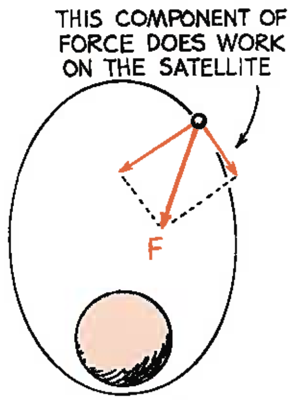


Fig. 12-12 In elliptical orbit, a component of force exists along the direction of the satellite's motion. This component changes the speed and, thus, the KE. (The perpendicular component changes only the direction.)

▶ **Questions**

1. The orbital path of a satellite is shown in the sketch in the previous question box (opposite page). In which marked positions *A* through *D* does the satellite have the greatest KE? Greatest PE? Greatest total energy?

2. How can the force of gravity change the speed of a satellite when it is in an elliptical orbit, but not when it is in a circular orbit?

▶ **Answers**

1. The KE is maximum at the perigee *A*; the PE is maximum at the apogee *C*; the total energy is the same everywhere in the orbit.

2. At any point on its path, the direction of motion of a satellite is always tangent to its path. If a component of force exists along this tangent, then the acceleration of the satellite will involve a change in speed as well as direction. In circular orbit the gravitational force is always perpendicular to the direction of motion of the satellite, just as every part of the circumference of a circle is perpendicular to the radius. So there is no component of gravitational force along the tangent, and only the direction of motion changes, not the speed. But when the satellite moves in directions that are not perpendicular to the force of gravity, as in an elliptical path, there is a component of force along the direction of motion which changes the speed of the satellite. From a work-energy point of view, a component of force along the direction the satellite moves does work to change its KE.

12.5 Escape Speed

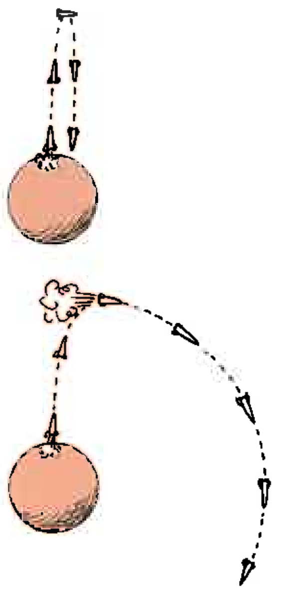


Fig. 12-13 The initial thrust of the rocket lifts it vertically. Another thrust tips it from its vertical course. When it is moving horizontally, it is boosted to the required speed for orbit.

When a payload is put into earth orbit by a rocket, the speed and direction of the rocket are important. For example, what would happen if the rocket were launched vertically and quickly achieved a speed of 8 km/s? Everyone had better get out of the way, because it would soon come crashing back at 8 km/s. To achieve orbit, the payload must be launched *horizontally* at 8 km/s once above the drag of the atmosphere. Launched only vertically, the old saying "What goes up must come down" becomes a sad fact of life.

But isn't there some vertical speed that is sufficient to insure that what goes up will escape and not come down? The answer is yes. Fire it at any speed greater than 11.2 km/s, and it will continually leave the earth, traveling more and more slowly, but will never be brought to a stop by earth gravity.* Let's look at this from an energy point of view.

How much work is required to move a payload against the force of earth gravity to a distance very, very far ("infinitely far") away? We might think that the PE would be infinite because the distance is infinite. But gravity diminishes with distance via the inverse-square law. The force of gravity is strong only close to the earth. Most of the work done in launching a rocket, for example, occurs near the earth's surface. It turns out that the value of PE for a 1-kilogram mass infinitely far away is 60 million joules (MJ). So to put a payload infinitely far from the earth's surface requires at least 60 MJ of energy per kilogram of load. We won't go through the calculation here, but a KE of 60 MJ corresponds to a speed of 11.2 km/s whatever the mass involved. This is the value of the **escape speed** from the surface of the earth.**

If we give a payload any more energy than 60 MJ/kg at the surface of the earth or, equivalently, any more speed than 11.2 km/s, then, neglecting air resistance, the payload will escape from the earth never to return. As it continues outward, its PE increases and its KE decreases. Its speed becomes less and less, though it

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* In a more-advanced physics course you would learn how the value of escape speed v, from any planet or any body, is given by $v = \sqrt{2GM/d}$, where G is the universal gravitational constant, M is the mass of the attracting body, and d is the distance from its center. (At the surface of the body d would simply be the radius of the body.)

** Interestingly enough, this might well be called the *maximum falling speed*. Any object, however far from earth, released from rest and allowed to fall to earth only under the influence of the earth's gravity would not exceed 11.2 km/s.

is never reduced to zero. The payload outruns the gravity of the earth. It escapes.

The escape speeds of various bodies in the solar system are shown in Table 12-1. Note that the escape speed from the sun is 620 km/s at the surface of the sun. Even at a distance equaling that of the earth's orbit, the escape speed from the sun is 42.5 km/s. The escape speed values in the table ignore the forces exerted by other bodies. A projectile fired from the earth at 11.2 km/s, for example, escapes the earth but not necessarily the moon, and certainly not the sun. Rather than recede forever, it will take up an orbit around the sun.

The first probe to escape the solar system, *Pioneer 10*, was launched from earth in 1972 with a speed of only 15 km/s. The escape was accomplished by directing the probe into the path of oncoming Jupiter. It was whipped about by Jupiter's great gravitational field, picking up speed in the process—just as the speed of a ball encountering an oncoming bat is increased when it departs from the bat. Its speed of departure from Jupiter was increased enough to exceed the sun's escape speed at the distance of Jupiter. *Pioneer 10* passed the orbit of Pluto in 1984. Unless it collides with another body, it will wander indefinitely through interstellar space. Like a note in a bottle cast into the sea, *Pioneer 10* contains information about the earth that might be of interest to extra-terrestrials, in hopes that it will one day wash up and be found on some distant "seashore."

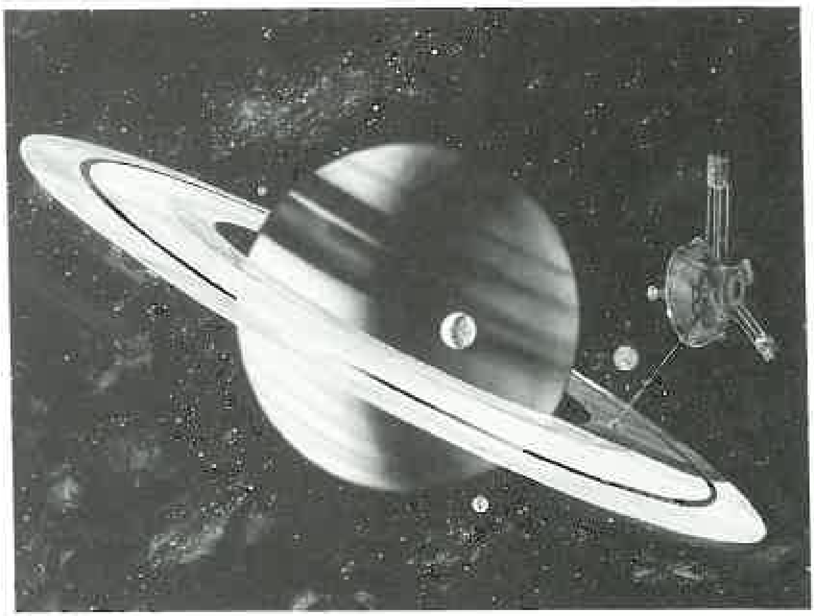

Fig. 12-14 *Pioneer 10*, launched from earth in 1972, escaped from the solar system in 1984 and is wandering in interstellar space.

Table 12-1 Escape Speeds at the Surface of Bodies in the Solar System

Astronomical Body	Mass (earth masses)	Radius (earth radii)	Escape speed (km/s)
Sun	330 000	109	620
Sun (at a distance of the earth's orbit)		23 000	42.5
Jupiter	318	11	61.0
Saturn	95.2	9	37.0
Neptune	17.3	3.4	25.4
Uranus	14.5	3.7	22.4
Earth	1.00	1.00	11.2
Venus	0.82	0.96	10.4
Mars	0.11	0.53	5.2
Mercury	0.05	0.38	4.3
Moon	0.01	0.27	2.4

It is important to point out that the escape speeds for different bodies refer to the initial speed given by a brief thrust, after which there is no force to assist motion. One could escape the earth at any sustained speed more than zero, given enough time. Suppose a rocket is going to a destination such as the moon. If the rocket engines burn out when still close to the earth, the rocket needs a minimum speed of 11.2 km/s. But if the rocket engines can be sustained for long periods of time, the rocket could go to the moon without ever attaining 11.2 km/s.

It is interesting to note that the accuracy with which an unpiloted rocket reaches its destination is not accomplished by staying on a preplanned path or by getting back on that path if it strays off course. No attempt is made to return the rocket to its original path. Instead, the control center in effect asks, "Where is it now with respect to where it ought to go? What is the best way to get there from here, given its present situation?" With the aid of high-speed computers, the answers to these questions are used in finding a *new* path. Corrective thrusters put the rocket on this new path. This process is repeated over and over again all the way to the goal.

Is there a lesson to be learned here? Suppose you find that you are "off course." You may, like the rocket, find it more fruitful to take a course that leads to your goal as best plotted from your present position and circumstances, rather than try to get back on the course you plotted from a previous position and under, perhaps, different circumstances.

12 Chapter Review

Concept Summary

An earth satellite is a projectile that moves fast enough horizontally so that it falls around the earth rather than into it.

- The speed of a satellite in a circular orbit is not changed by gravity.
- The speed of a satellite in an elliptical orbit increases as the distance from the earth decreases, and vice versa.
- The sum of the kinetic and potential energies is constant all through the orbit.
- If something is launched from earth with a great enough energy per kilogram of mass, it will move so fast that it will never return to earth.

Important Terms

apogee (12.4)
ellipse (12.3)
escape speed (12.5)
focus (12.3)
perigee (12.4)
period (12.2)

Review Questions

1. If you simply drop a stone from a position of rest, how far will it fall vertically in the first second? If you instead move your hand sideways and drop it (throw it), how far will it fall vertically in the first second? (12.1)

2. What do the distances 8000 m and 4.9 m have to do with a line tangent to the earth's surface? (12.1)

3. How does a satellite in circular orbit move in relation to the surface of the earth? (12.2)

4. Why doesn't gravitational force change the speed of a satellite in circular orbit? (12.2)

5. a. What is the period of a satellite in close circular orbit about the earth?
   b. Is the period greater or less for greater distances from the earth? (12.2)

6. What is an ellipse? (12.3)

7. Why does gravitational force change the speed of a satellite in elliptical orbit? (12.4)

8. a. At what part of an elliptical orbit is the speed of a satellite maximum?
   b. Where is it minimum? (12.4)

9. The sum of PE and KE for a satellite in a circular orbit is constant. Is this sum also constant for a satellite in an elliptical orbit? (12.4)

10. Why does the force of gravity do no work on a satellite in circular orbit, but does do work on a satellite in an elliptical orbit? (12.4)

11. What will be the fate of a projectile that is fired vertically at 8 km/s? At 12 km/s? (12.5)

12. Why does most of the work done in launching a rocket take place when the rocket is still close to the earth's surface? (12.5)

13. a. How fast would a particle have to be ejected from the sun to leave the solar system?
    b. What speed would be needed if it started at a distance from the sun equal to the earth's distance from the sun? (12.5)

14. What is the escape speed from the moon? (12.5)

15. Although the escape speed from the surface of the earth is said to be 11.2 km/s, couldn't a rocket with enough fuel escape at any speed? Defend your answer. (12.5)

Activity

Draw an ellipse with a loop of string, two tacks, and a pen or pencil. Try different tack spacings for a variety of ellipses.

Think and Explain

1. A satellite can orbit at a 5-km altitude above the moon, but not at 5 km above the earth. Why?

2. Does the speed of a satellite around the earth depend on its mass? Its distance from the earth? The mass of the earth?

3. If a cannonball is fired from a tall mountain, gravity changes its speed all along its trajectory. But if it is fired fast enough to go into circular orbit, gravity does not change its speed at all. Why is this so?

4. If you stopped an earth satellite dead in its tracks, it would simply crash into the earth. Why, then, don't the communications satellites that "hover motionless" above the same spot on earth simply crash into the earth?

5. Would you expect the speed of a satellite in close circular orbit about the moon to be less than, equal to, or greater than 8 km/s? Defend your answer.

6. Why do you suppose that a space shuttle is sent into orbit by firing it in an easterly direction (the direction in which the earth spins)?

7. If an astronaut in an orbiting space shuttle wished to drop something to earth, how could this bé accomplished?

8. What is the maximum possible speed of impact upon the surface of the earth for a faraway object initially at rest that falls to earth by virtue of only the earth's gravity?

9. Is it possible for a rocket to escape from earth without ever traveling as fast as 11.2 km/s? Explain.

10. If the earth somehow became more massive, would the escape speed be less than, equal to, or more than 11.2 km/s? Defend your answer.