6 Vectors

Suppose by chance you found yourself sitting beside a physicist when taking a long bus ride. Suppose the physicist was in a talkative mood and told you about some of the things he or she did for a living. To explain things, your companion would likely doodle on a scrap of paper. Physicists love using doodles to explain ideas. Einstein was famous for that. You'd see some arrows in the doodles. The arrows would probably represent the *magnitude* (how much) and the *direction* (which way) of a certain quantity. The quantity might be the electric current that operates a minicomputer, or the orbital velocity of a communications satellite, or the enormous force that lifts an Atlas rocket off the ground. Whenever the length of an arrow represents the magnitude of a quantity, and the direction of the arrow represents the direction of the quantity, the arrow is called a **vector**.

6.1 Vector and Scalar Quantities

Some quantities require both magnitude and direction for a complete description. These are called **vector quantities**. A force, for example, has a direction as well as a magnitude. So does a velocity. Force and velocity are the most familiar vector quantities, but there are a few others treated in later chapters.

Many quantities in physics, such as mass, volume, and time, can be completely specified by their magnitudes. They do not involve any idea of direction. These are called **scalar quantities**. They obey the ordinary laws of addition, subtraction, multiplication, and division. If 3 kg of sand is added to 1 kg of cement, the resulting mixture has a mass of 4 kg. If 5 liters of water is poured from a pail which initially had 8 liters of water in it, the resulting volume is 3 liters. If a scheduled 1-hour trip runs into a 15-minute delay, the trip ends up taking $1\frac{1}{4}$ hours. In each of these cases, no direction is involved. We see that 10 kilograms north, 5 liters east, or 15 minutes south have no meaning. Quantities that involve only magnitude and not direction are scalars.

6.2 Vector Representation of Force

It is easy to draw a vector that represents a force. The length, drawn to some suitable scale, indicates the magnitude of the force. The orientation on the paper and the arrowhead show the direction.

Figure 6-1 left shows a horse pulling a cart and a man pushing the cart from behind. The diagram shows vectors for these two forces acting on the cart. The horse applies twice as much force on the cart as the man. So the vector for the force supplied by the horse is twice as long as the one for the force supplied by the man. The vectors have been drawn to a scale on which 1 cm represents 100 N. The vectors are pointing in the same direction, since the two forces are in the same direction.

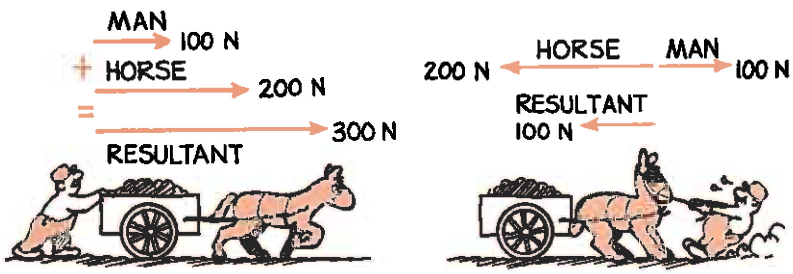

Fig. 6-1 The resultant of two forces depends on the directions of the forces as well as on their magnitudes.

The man pushes with 100 N and the horse pulls with 200 N. Since the two forces act in the same direction, the resulting pull is equal to the sum of the individual pulls and acts in the same direction. The cart moves as if both forces were replaced by a single net force of 300 N. This net force is called the **resultant** of the two forces. We see it is represented by a vector 3 cm long.

Now suppose that the horse is pushing backwards with a force of 200 N while the man is pulling forward with a force of 100 N (Figure 6-1 right). The two forces then act in opposite directions. The resultant (net force) is equal to the difference between them, 200 N − 100 N = 100 N, and acts in the direction of the larger force. It is represented by a vector 1 cm long.

6.3 Vector Representation of Velocity

Speed is a measure of how fast something is moving; it can be in any direction. When we take into account the direction of motion as well as the speed, we are talking about velocity. Velocity, like force, is a vector quantity.

Consider an airplane flying due north at 100 km/h relative to the surrounding air. There is a tailwind (wind from behind) that also moves due north at a velocity of 20 km/h. This example is represented with vectors in Figure 6-2 left. Here the velocity vectors are scaled so that 1 cm represents 20 km/h. Thus, the 100-km/h velocity of the airplane is shown by the 5-cm-long vector and the 20-km/h tailwind is shown by the 1-cm-long vector. You can see (with or without the vectors) that the resultant velocity is going to be 120 km/h. Without the tailwind, the airplane would travel 100 km in one hour relative to the ground below. With the tailwind, it would travel 120 km in one hour.

Suppose, instead, that the wind is a headwind (wind head-on), so that the airplane flies into the wind rather than with the wind. Now the velocity vectors are in opposite directions (Figure 6-2 right). Their resultant is 100 km/h − 20 km/h = 80 km/h. Flying against a 20-km/h headwind, the airplane would travel only 80 km relative to the ground in one hour.

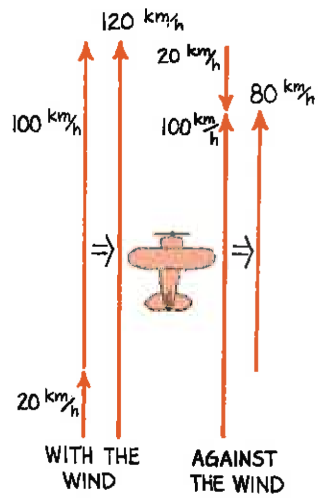

Fig. 6-2 The velocity of an airplane relative to the ground depends on its velocity relative to the air and on the wind velocity.

► **Questions**

1. In the cart, horse, and man example shown in Figure 6-1 left, suppose the man's kid brother assists by also pushing forward on the cart, but with a force of 50 N. What would then be the resultant force the men and horse exert on the cart?
2. Suppose in the previous question that all parties exert forces as stated in a headwind of 10 km/h. What would then be the resultant force the men and horse exert on the cart?

► **Answers**

(A reminder: are you reading this before you have thought about the questions and come up with your own answers? Finding, seeing, and remembering the answer is *not* the way to do physics. Learning to *think* about the ideas of physics is more important. Think first, then look! It will make a difference.)

1. The resultant will be the sum of the applied forces: 200 N + 100 N + 50 N = 350 N.
2. The resultant force is the same, 350 N. Be careful here. Adding apples to a pile of oranges doesn't increase the number of oranges. Similarly, we can't add or subtract velocity from force.

6.4 Geometric Addition of Vectors

Consider the forces exerted by the horses towing the barge in Figure 6-3 left. When vectors act at an angle to each other, a simple geometrical technique can be used to find the magnitude and direction of the resultant. The two vectors to be added are drawn with their tails touching (see Figure 6-3 right). A projection of each vector is drawn (dashed lines) starting at the head of the other vector. The four-sided shape that results is known as a *parallelogram* because the opposite sides are parallel and of equal length.* The resultant of the two forces is the diagonal of the parallelogram between the points where the tails meet and the dashed lines meet.**

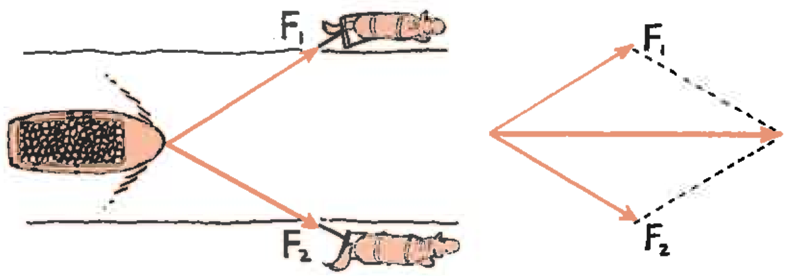

Fig. 6-3 The barge moves under the action of the resultant of the two forces F_1 and F_2. The direction of the resultant is along the diagonal of the parallelogram constructed with sides F_1 and F_2.

We can see that the barge will not move in the direction of either of the forces exerted by the horses, but rather in the direction of their resultant. The resultant is found by using the following rule for the addition of vectors:

> The resultant of two vectors may be represented by the diagonal of a parallelogram constructed with the two vectors as sides.

You can apply this rule to other pairs of forces that act on a common point. Figure 6-4 shows forces of 3 N to the north and

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* When the angles in a parallelogram are 90°, it becomes a rectangle; if the four sides of the rectangle are the same length, it is a square.

** Since vector arrows only *represent* forces, it doesn't matter where you place them on a drawing so long as their directions and lengths are correct. Another way to find their resultant is to rearrange the arrows in any order so they are united tail to tip. A new vector, drawn from the tail of the first vector to the tip of the last vector, represents the resultant or net force.

4 N to the east. Using a scale of 1 N:1 cm, you construct a parallelogram, using the vectors as sides. Since the vectors are at right angles, your parallelogram is simply a rectangle. If you draw a diagonal from the tails of the vector pair, you have the resultant. Measure the length of the diagonal and refer to the scale, and you have the magnitude of the resultant. The angle can be found with a protractor.

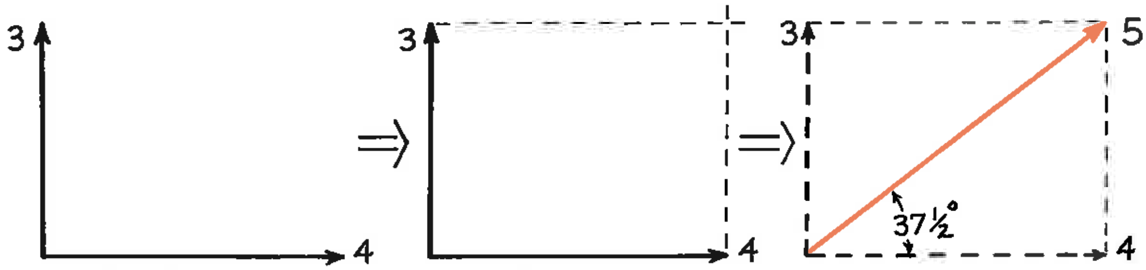

Fig. 6-4 The 3-N and 4-N forces add to produce a force of 5 N.

▶ **Exercises**

1. By the parallelogram method construct the resultants of the 3-N and 4-N forces represented by the vectors shown. They are drawn to a scale on which 1 cm:1 N. Measure your resultants with a ruler and compare them to the correct answers given at the bottom of the page.

2. What are the minimum and maximum resultants possible for a 3-N and a 4-N force acting on the same object?

With the technique of vector addition you can correct for the effect of a crosswind on the velocity of an airplane. Consider a slow-moving airplane that flies north at 80 km/h and is caught in a strong crosswind of 60 km/h blowing east. Figure 6-5 shows vectors for the airplane velocity and wind velocity. The scale

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▶ **Answers**

1. Left: 6 N; right: 4 N.

2. The minimum resultant occurs when the forces oppose each other: 4 N − 3 N = 1 N. The maximum resultant occurs when they are in the same direction: 4 N + 3 N = 7 N. (At angles to each other, 3 N and 4 N can combine to range between 1 N and 7 N.)

here is 1 cm:20 km/h. The diagonal of the constructed parallelogram (rectangle in this case) measures 5 cm, which represents 100 km/h. So the airplane moves at 100 km/h relative to the ground, in a northeasterly direction.*

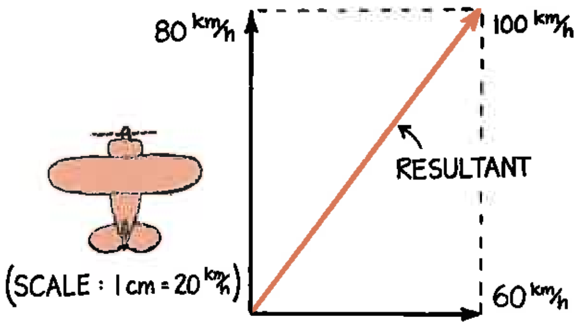

Fig. 6-5 An 80-km/h airplane flying in a 60-km/h crosswind has a resultant speed of 100 km/h relative to the ground.

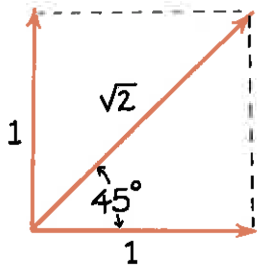

Fig. 6-6 The diagonal of a square is $\sqrt{2}$ the length of one of its sides.

There is a special case of the parallelogram that often occurs. When two vectors that are equal in magnitude and at right angles to one another are to be added, the parallelogram becomes a square. Since for any square the length of a diagonal is $\sqrt{2}$, or 1.414, times one of the sides, the resultant is $\sqrt{2}$ times one of the vectors. For example, the resultant of two equal vectors of magnitude 100 acting at right angles to each other is 141.4.

6.5 Equilibrium

The method of combining vectors by the parallelogram rule is an experimental fact. It can be shown to be correct by considering an example that is common and quite surprising the first time—the case of being able to hang safely from a vertical clothesline but not being able to do so when the line is strung horizontally. Invariably, it breaks (Figure 6-7).

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* Whenever the vectors are at right angles to each other, their resultant can be found by the Pythagorean Theorem, a well-known tool of geometry. It states that the square of the hypotenuse of a right-angle triangle is equal to the sum of the squares of the other two sides. Note that two right triangles are present in the parallelogram (rectangle in this case) in Figure 6-5. From either one of these triangles we get:

$$\begin{aligned} \text{resultant}^2 &= (60\ \text{km/h})^2 + (80\ \text{km/h})^2 \\ &= 3600\ (\text{km/h})^2 + 6400\ (\text{km/h})^2 \\ &= 10\,000\ (\text{km/h})^2 \end{aligned}$$

The square root of 10 000 $(\text{km/h})^2$ is 100 km/h, as expected.

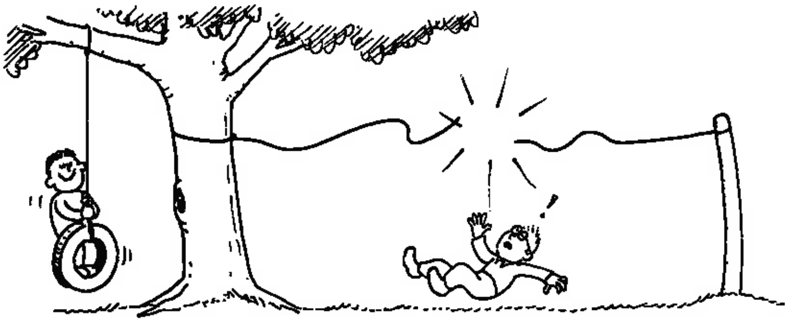


Fig. 6-7 You can safely hang from a piece of clothesline when it hangs vertically, but you'll break it if you attempt to make it support your weight when it is strung horizontally.

We can understand this with spring scales that are used to measure weight. Consider a block that weighs 10 N, about the weight of ten apples. If we suspend it from a single scale, as in Figure 6-8 left, the reading will be its weight, 10 N. Did you know that if you stand with your weight evenly divided on two bathroom scales, each scale will record half your weight? This is because the springs in the scales are compressed and, in effect, push up on you just as hard as gravity pulls down on you. If two scales support your weight, each will have half the job. The same is true if you hang by a pair of scales. In this case the springs that support you are stretched, and each scale has half the job and reads half your weight. So if we suspend the 10-N block from a pair of vertical scales (Figure 6-8 right), each scale will read 5 N. The scales pull up with a resultant force that equals the weight of the block. The diagram shows a pair of 5-N vectors that have a 10-N resultant that exactly opposes the 10-N weight vector. The net force on the block is zero, and the block hangs at rest; we say it is in **equilibrium**. The key idea is this: if a 10-N block is to hang in equilibrium, the resultant of the forces supplied by the pair of springs must equal 10 N. For vertical orientation this is easy; 5 N + 5 N = 10 N. This is all Chapter 4 stuff. ← ? stuff ?

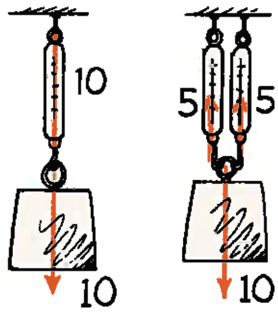


Fig. 6-8 (Left) When a 10-N block hangs vertically from a single spring scale, the scale pulls upward with a force of 10 N. (Right) When it hangs vertically from two spring scales, each scale pulls upward with a force of half the weight, or 5 N.

Now let's look at a non-vertical arrangement. In Figure 6-9 left, we see that when the supporting spring scales hang at an angle to support the block, the springs are stretched more, as indicated by the greater reading. At 60° from the vertical, the readings are 10 N each—double what they were when the scales were hanging vertically! Can you see the explanation? The resultant of the two scale readings must be 10 N upward to balance the downward weight of the block. A pair of 5-N vectors will produce a 10-N resultant only if they are parallel and acting in the same direction. If the vectors have different directions, then each vector must be greater than 5 N to produce a resultant of 10 N. For equilibrium, the diagonal of the parallelogram formed by the vector sides, whatever the angle between them, must remain the same. Why? Because the diagonal must correspond to 10 N, exactly equal and opposite to the 10 N weight of the block.

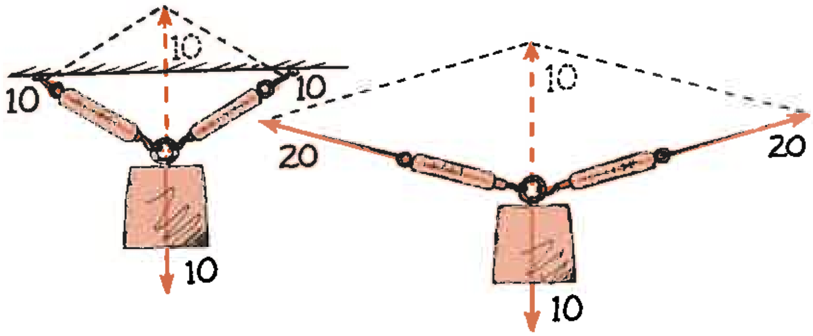
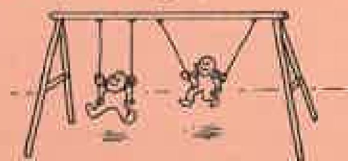


Fig. 6-9 As the angle between the spring scales increases, the scale readings increase so that the resultant (dashed-line vector) remains at 10 N upward, which is required to support the 10-N block.

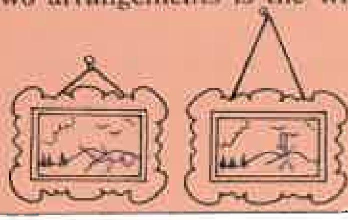
In Figure 6-9 right, where the angle from the vertical has been increased to 75.5°, each spring must pull with 20 N to produce the required 10-N resultant. As the angle between the scales is increased, the scale readings increase. Can you see that as the angle between the sides of the parallelogram increases, the magnitude of the sides must increase if the diagonal is to remain the same? If you understand this, you understand why you can't be supported by a horizontal clothesline without producing a stretching force that is considerably greater than your weight. The parallelogram rule turns out to be quite interesting.

▶ **Questions**

1. If the kids on the swings are of equal weight, which swing is more likely to break?



2. Two pictures of equal weight are hung in a gallery as shown. In which of the two arrangements is the wire more likely to break?



▶ **Answers**

1. The tension is greater in the ropes that hang at an angle, so they are more likely to break than the vertical ropes.
2. The tension is greater in the picture to the left, because the supporting rope makes a greater angle with respect to the vertical than the picture on the right. This is similar to the stretched clothesline.

6.6 Components of Vectors

Two vectors acting on the same object may be replaced by a single vector (the resultant) that produces the same effect upon the object as the combined effects of the given vectors. The reverse is also true: any single vector may be regarded as the resultant of two vectors, each of which acts on the body in some direction other than that of the given vector. These two vectors are known as the **components** of the given vector that they replace. The process of determining the components of a vector is called **resolution**.

A man pushing a lawnmower applies a force that pushes the machine forward and also against the ground. In Figure 6-10, vector F represents the force applied by the man. We can separate this force into two components. Vector Y is the vertical component, which is the downward push against the ground. Vector X is the horizontal component, which is the forward force that moves the lawnmower.

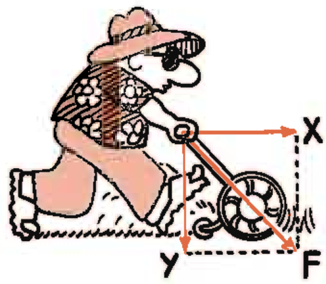


Fig. 6-10 The force F applied to the lawnmower may be resolved into a horizontal component, X, and a vertical component, Y.

We can find the magnitude of these components by drawing a rectangle with F as the diagonal. Since X and Y are the sides of a parallelogram, vector F is the resultant of the vectors X and Y. Hence the two components X and Y acting together are equivalent to the force F. That is, the motion of the lawnmower is the same whether we assume that the man exerts two forces, components X and Y, or only one force, F.

The rule for finding the vertical and horizontal components of any vector is relatively simple, and is illustrated in Figure 6-11. A vector V is drawn in the proper direction to represent the force, velocity, or whatever vector is in question (Figure 6-11 left). Then vertical and horizontal lines are drawn at the tail of the vector (Figure 6-11 right). A rectangle is drawn that encloses the vector V in such a way that V is a diagonal and the sides of the rectangle are the desired components. We see that the components of the vector V are then represented in direction and magnitude by the vectors X and Y.

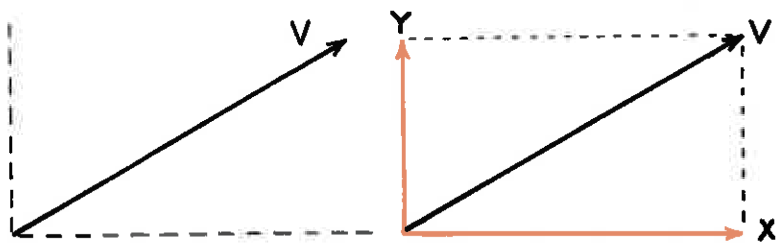


Fig. 6-11 The vector V has component vectors X and Y.

Any vector can be represented by a pair of components that are at right angles to each other. This is neatly illustrated in the explanation of a sailboat sailing against the wind. (See Appendix C, Vector Applications, at the back of this book.)

► **Exercise**

With a ruler, draw the horizontal and vertical components of the two vectors shown. Measure the components and compare your findings with the answers given at the bottom of the page.

6.7 Components of Weight

We all know that a ball will roll faster down a steep hill than a hill with a small slope. The steeper the hill is, the greater the acceleration of the ball. We can understand why this is so with vector components. The force of gravity that acts on things gives them weight, which we represent as vector W. The force vector W acts only straight down—toward the center of the earth—but components of W may act in any direction. It is most often useful to consider components that are at right angles to each other.

In Figure 6-12 we see W broken up into components A and B, where A is parallel to the surface and B is perpendicular to the surface. It is component A that makes the ball move. Component B presses the ball against the surface. Pictures are better than words, so study the figure and see how the magnitudes of the components vary for different slopes.

► **Answer**

Left vector: the horizontal component is 3 cm; the vertical component is 4 cm. Right vector: the horizontal component is 6 cm; the vertical component is 4 cm.

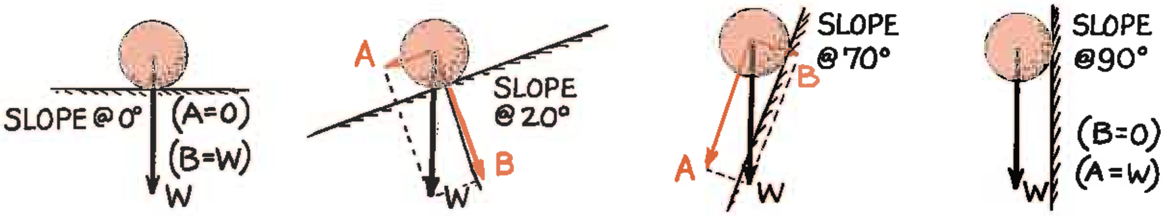


Fig. 6-12 The weight of the ball is represented by vector *W*, which has perpendicular components *A* and *B*. Vector *A* serves to change the speed of the ball, while vector *B* presses it against the surface. Note how the magnitudes of *A* and *B* vary from zero to *W*.

Can you see that only when the slope is zero—when the surface is horizontal—is component *A* equal to zero? That's why the speed of the ball does not change on a horizontal surface. Note another thing when the surface is horizontal. Component *B* is equal to *W*; the ball presses against the surface with the most force. But when the slope is 90°, component *B* becomes zero and *A* equals *W*, so the ball has its maximum acceleration.

▶ **Question**

At what angle will components *A* and *B* in Figure 6-12 have equal magnitudes? At what angle will *A* equal *W*? At what angle will *A* be greater in magnitude than *W*?

6.8 Projectile Motion

Chapter 2 discussed the vector quantities velocity and acceleration. Since only horizontal and vertical motion was considered, we did not need to know about vector addition or the techniques of vector resolution. But for objects projected at angles other than straight up or straight down, we do.

A **projectile** is any object that is projected by some means and continues in motion by its own inertia. A cannonball shot from a cannon, a stone thrown into the air, or a ball that rolls off the edge of the table are all projectiles. These projectiles follow curved paths that at first thought seem rather complicated. However, these paths are surprisingly simple when we look at the horizontal and vertical components of motion separately.

▶ **Answer**

Components *A* and *B* have equal magnitudes at 45°; $A = W$ at 90°; *A* cannot have a greater magnitude than *W* at any angle.

The horizontal component of motion for a projectile is no more complex than the horizontal motion of a bowling ball rolling freely along a level bowling alley. If the retarding effect of friction can be ignored, the bowling ball moves at constant velocity. It covers equal distances in equal intervals of time. It rolls of its own inertia, with no component of force acting in its direction of motion. It rolls without accelerating. The horizontal part of a projectile's motion is just like the bowling ball's motion along the alley (Figure 6-13 left).

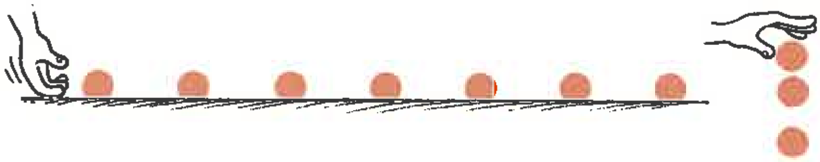


Fig. 6-13 (Left) Roll a ball along a level surface, and its velocity is constant because no component of gravitational force acts horizontally. (Right) Drop it, and it accelerates downward and covers greater vertical distances each second.

The vertical component of motion for a projectile following a curved path is just like the motion described in Chapter 2 for a freely-falling object. Like a ball dropped in mid-air, the projectile moves in the direction of earth gravity and accelerates downward (Figure 6-13 right). The increase in speed in the vertical direction causes successively greater distances to be covered in each successive equal-time interval.

Interestingly enough, the horizontal component of motion for a projectile is completely independent of the vertical component of motion. Each acts independently of the other. Their combined effects produce the variety of curved paths that projectiles follow.

The multiple-flash exposure of Figure 6-14 shows equally-timed successive positions for a ball rolled off a horizontal table. Investigate the photo carefully, for there's a lot of good physics there. The curved path of the ball is best analyzed by considering the horizontal and vertical components of motion separately. There are two important things to notice. The first is that the ball's horizontal component of motion doesn't change as the falling ball moves sideways. The ball travels the same horizontal distance in the equal times between each flash. That's because there is no component of gravitational force acting horizontally. Gravity acts only downward, so the only acceleration of the ball is downward. The second thing to note from the photo is that the vertical positions become farther apart with time. The distances traveled vertically are the same as if the ball were simply dropped. It is interesting to note that the downward motion of the ball is the same as that of free fall.

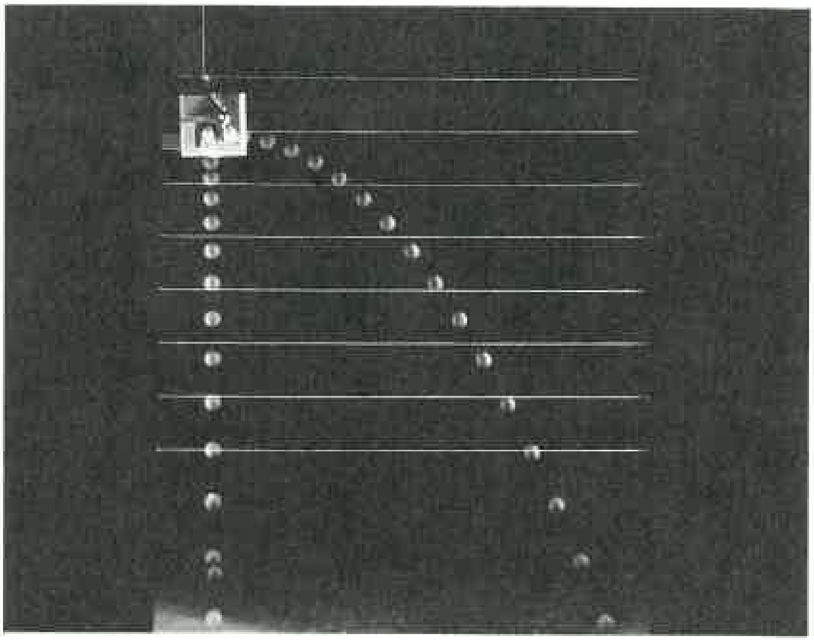


Fig. 6-14 A multiple-flash photograph of a ball rolling off a horizontal table. Notice that in equal times it travels equal horizontal distances but increasingly greater distances vertically. Do you know why?

The path traced by a projectile that accelerates only in the vertical direction while moving at a constant horizontal velocity is called a *parabola*. When air resistance can be neglected—usually for slow-moving projectiles or ones very heavy compared to the forces of air resistance—the curved paths are *parabolic*.

► **Question**

At the instant a horizontally held rifle is fired over a level range, a bullet held at the side of the rifle is released and drops to the ground. Which bullet—the one fired downrange or the one dropped from rest—strikes the ground first?

► **Answer**

Both bullets fall the same vertical distance with the same acceleration g due to gravity and therefore strike the ground at the same time. Can you see that this is consistent with our analysis of Figure 6-14? We can reason this another way by asking which bullet would strike the ground first if the rifle were pointed at an upward angle. In this case, the bullet that is simply dropped would hit the ground first. Now consider the case where the rifle is pointed downward. The fired bullet hits first. So upward, the dropped bullet hits first; downward, the fired bullet hits first. There must be some angle at which there is a dead heat—where both hit at the same time. Can you see it would be when the rifle is neither pointing upward nor downward—when it is horizontal?

6.9 Upwardly Moving Projectiles

Consider a cannonball shot at an upward angle. Pretend for a moment that there is no gravity; then according to the law of inertia, the cannonball will follow the straight-line path shown by the dashed line in Figure 6-15. But there *is* gravity, so this doesn't happen. What really happens is that the cannonball continually *falls beneath this imaginary line* until it finally strikes the ground. Get this: The vertical distance it falls beneath any point on the dashed line is the same vertical distance it would fall if it were dropped from rest and had been falling for the same amount of time. This distance, as introduced in Chapter 2, is given by $d = \frac{1}{2}gt^2$, where t is the elapsed time.

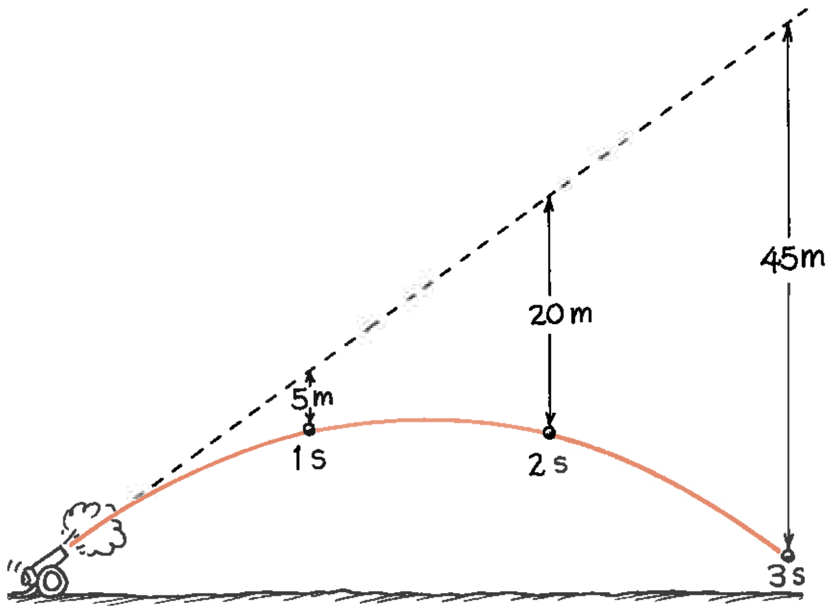


Fig. 6-15 With no gravity the projectile would follow the straight-line path (dashed line). But because of gravity, it falls beneath this line the same vertical distance it would fall if released from rest. Compare the distances fallen with Table 2-3 in Chapter 2. (With $g = 9.8\ \text{m/s}^2$, these distances are more accurately 4.9 m, 19.6 m, and 44.1 m.)

We can put this another way: Shoot a projectile skyward at some angle and pretend there is no gravity. After so many seconds t, it should be at a certain point along a straight-line path. But because of gravity, it isn't. Where is it? The answer is, it's directly below this point. How far below? The answer in meters is $5t^2$ (or more accurately, $4.9t^2$). Isn't that neat?

Note another thing from Figure 6-15. The cannonball moves equal horizontal distances in equal time intervals. That's because no acceleration takes place horizontally. The only acceleration is vertically, in the direction of earth gravity. The vertical distance it falls below the imaginary straight-line path during equal time intervals continually increases with time.

Figure 6-16 shows vectors representing both horizontal and vertical components of velocity for a projectile following a parabolic path. Notice that the horizontal component is everywhere the same, and only the vertical component changes. Note also that the actual velocity is represented by the vector that forms the diagonal of the rectangle formed by the vector components. At the top of the path the vertical component vanishes to zero, so the actual velocity there *is* the horizontal component of velocity at all other points. Everywhere else the magnitude of velocity is greater (just as the diagonal of a rectangle is greater than either of its sides).

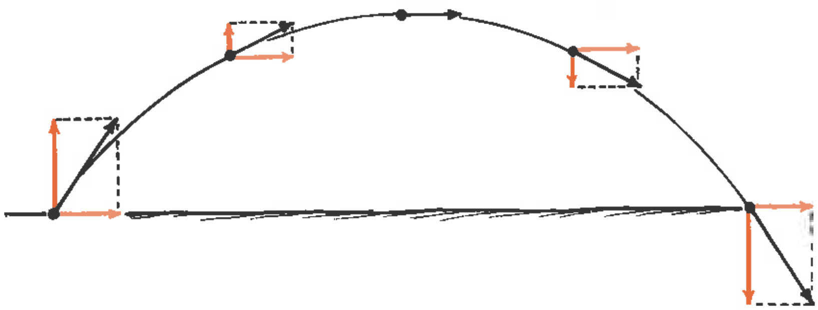


Fig. 6-16 The velocity of a projectile at various points along its path. Note that the vertical component changes and the horizontal component is the same everywhere.

Figure 6-17 shows the path traced by a projectile with the same launching speed at a steeper angle. Notice that the initial velocity vector has a greater vertical component than when the projection angle is less. This greater component results in a higher path. But the horizontal component is less so the range is less.

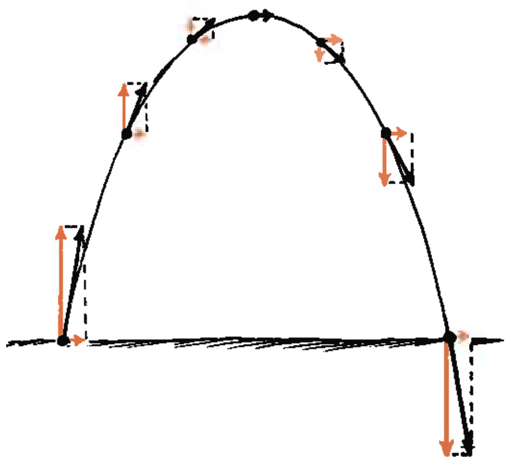


Fig. 6-17 Path for a steeper projection angle.

Figure 6-18 shows the paths of several projectiles all having the same initial speed but different projection angles. The figure neglects the effects of air resistance, so the paths are all parabolas. Notice that these projectiles reach different *altitudes*, or heights above the ground. They also have different *horizontal ranges*, or distances traveled horizontally.

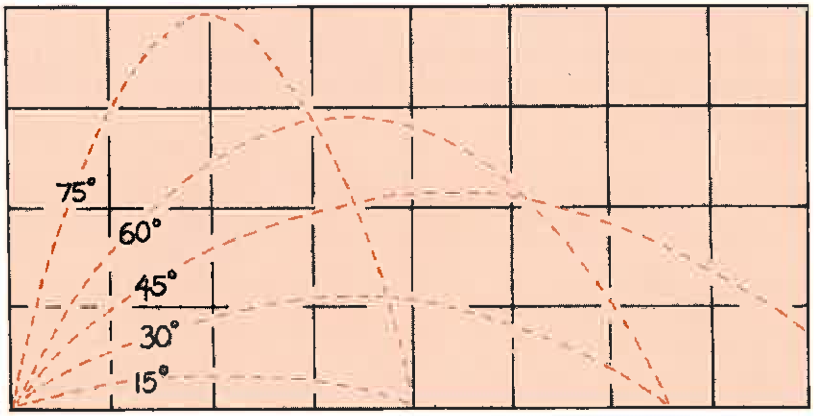


Fig. 6-18 Ranges of a projectile shot at the same speed at different projection angles.

The remarkable thing to note from Figure 6-18 is that the same range is obtained from two different projection angles—angles that add up to 90 degrees! An object thrown into the air at an angle of 60 degrees, for example, will have the same range as if it were thrown at the same speed at an angle of 30 degrees. For the smaller angle, of course, the object remains in the air for a shorter time.

Fig. 6-19 Maximum range is attained when the ball is batted at an angle of nearly 45°. (In cases where the weight of the projectile is comparable to the applied force, as when a heavy javelin is thrown, the applied force does not produce the same speed for different projection angles, and maximum range occurs for angles quite a bit less than 45°.)

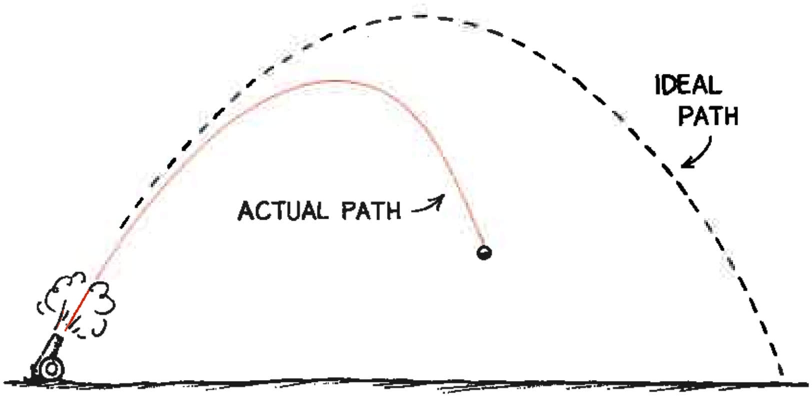


Fig. 6-20 In the presence of air resistance, the path of a high-speed projectile falls short of a parabola (dashed curve).

We have emphasized the special case of projectile motion without air resistance. When there is air resistance, the range of a projectile is somewhat shorter and is not a true parabola (Figure 6-20).

> ▶ **Questions**
> 1. A projectile is shot at an angle into the air. If air resistance is negligible, what is its downward acceleration? Its horizontal acceleration?
> 2. At what part of its path does a projectile have minimum speed?

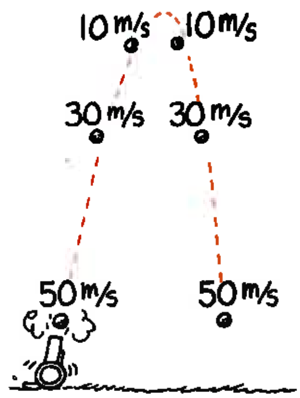


Fig. 6-21 Without air resistance, speed lost while going up equals speed gained while coming down; time up equals time down.

If air resistance is small enough to be negligible, a projectile will rise to its maximum height in the same time it takes to fall from that height to the ground. This is because its deceleration by gravity while going up is the same as its acceleration by gravity while coming down. The speed it loses while going up is therefore the same as the speed it gains while coming down. So the projectile arrives at the ground with the same speed it had when it was projected from the ground.

If an object is projected fast enough so that its curvature matches the curvature of the earth, and it is above the atmosphere so that air resistance does not affect its motion, it will fall all the way around the earth and be an earth satellite. This interesting topic is treated in Chapter 12.

▶ **Answers**

1. Its downward acceleration is g because the force of gravity is downward; its horizontal acceleration is zero because no horizontal forces act on it.
2. The speed of a projectile is minimum at the top of its path. If it is launched vertically, its speed at the top is zero. If it is projected at an angle, the vertical component of velocity is zero at the top, leaving only the horizontal component. So the speed at the top is equal to the horizontal component of the projectile's velocity at any point. Isn't that neat?

?

6 Chapter Review

Concept Summary

Vector quantities have both magnitude and direction.

- A vector is an arrow whose length represents the magnitude of a vector quantity and whose direction represents the direction of the quantity.

The resultant of several forces or several velocities can be determined from a vector diagram drawn to scale.

- When something is in equilibrium, the resultant of all the forces supporting it must exactly oppose its weight.

Any single vector can be replaced by two components that add to form the original vector.

- It is often convenient to study the horizontal and vertical components of forces or velocities.
- When gravity is the only force acting on a projectile, the horizontal component of its velocity does not change.

Important Terms

component (6.6)	resultant (6.2)
equilibrium (6.5)	scalar quantity (6.1)
projectile (6.8)	vector (6.1)
resolution (6.6)	vector quantity (6.1)

Review Questions

1. How does a vector differ from a scalar? (6.1)

2. If a vector that is 1 cm long represents a force of 5 N, how many newtons does a vector 2 cm long, drawn to the same scale, represent? (6.2)

3. a. What is the resultant of a pair of forces, 100 N upward and 75 N downward?
   b. What is their resultant if they both act downward? (6.2)

4. Why is speed classified as a scalar, and velocity as a vector? (6.3)

5. What is the resultant velocity of an airplane that normally flies at 200 km/h if it experiences a 50-km/h tailwind? A 50-km/h headwind? (6.3)

6. What is a parallelogram? (6.4)

7. When a parallelogram is constructed in order to add forces, what represents the resultant of the forces? (6.4)

8. What is the magnitude of the resultant of two vectors of magnitudes 4 and 3 that are at right angles to each other? (6.4)

9. What is the magnitude of the resultant of a pair of 100-N vectors that are at right angles to each other? (6.4)

10. The tension in a clothesline carrying a load of wash is appreciably greater when the clothesline is strung horizontally than when it is strung vertically. Why? (6.5)

11. What is the net force, or equivalently, the resultant force that acts on an object when it is in equilibrium? (6.5)

12. Compared to your weight, what is the stretching force in your arm when you let yourself hang motionless by one arm? By both arms vertically? Is this force greater or less if you hang with your hands wide apart? Why? (6.5)

13. Distinguish between the method of geometric addition of vectors and vector resolution. (6.4, 6.6)

14. What are the magnitudes of the horizontal and vertical components of a vector that is 100 units long, and oriented at 45°? (6.6)

15. The weight of a ball rolling down an inclined plane can be broken into two vector components: one acting parallel to the plane, and the other acting perpendicular to the plane.
   a. At what slope angle are these two components equal?
   b. At what slope angle is the component parallel to the plane equal to zero?
   c. At what slope angle is the component parallel to the plane equal to the weight? (6.7)

16. Why does a bowling ball move without acceleration when it rolls along a bowling alley? (6.8)

17. In the absence of air resistance, why does the horizontal component of velocity for a projectile remain constant, and why does only the vertical component change? (6.8)

18. How does the downward component of the motion of a projectile compare to the motion of free fall? (6.8)

19. At the instant a ball is thrown horizontally over a level range, a ball held at the side of the first is released and drops to the ground. If air resistance can be neglected, which ball—the one thrown or the one dropped from rest—strikes the ground first? (6.8)

20. a. How far below an initial straight-line path will a projectile fall in one second?
   b. Does your answer depend on the angle of launch or on the initial speed of the projectile? Defend your answer. (6.9)

21. a. A projectile is fired straight upward at 100 m/s. How fast is it moving at the instant it reaches the top of its trajectory?
   b. What is the answer if the projectile is fired upward at 45° instead? (6.9)

22. At what angle should a slingshot be oriented for maximum altitude? For maximum horizontal range? (6.9)

23. Neglecting air resistance, if you throw a ball straight upward with a speed of 20 m/s, how fast will it be moving when you catch it? (6.9)

24. a. Neglecting air resistance, if you throw a baseball at 20 m/s to your friend who is on first base, will the catching speed be greater than, equal to, or less than 20 m/s?
   b. How about if air resistance *is* a factor? (6.9)

Activity

Place a coin at the edge of a smooth table so that it overhangs slightly. Then place a second coin on the table top some distance from the overhanging coin. Set the second coin sliding across the table (such as by snapping it with your finger) so that it strikes the overhanging coin and both coins fall to the floor below. Which—if either—hits the floor first? Does your answer depend on the speed of the sliding coin?

Think and Explain

1. a. What is the maximum possible resultant of a pair of vectors, one of magnitude 5 and the other of magnitude 4?
   b. What is the minimum possible resultant?

2. A boat is rowed at 8 km/h directly across a river that flows at 6 km/h (Figure A).
   a. What is the resultant speed of the boat?
   b. How fast and in what direction can the boat be rowed to reach a destination directly across the river?

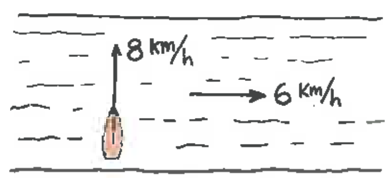


Fig. A

3. By whatever means, find the direction of the airplane in Figure 6-5.

4. In which position is the tension the least in the arms of the weightlifter shown in Figure B? The most?

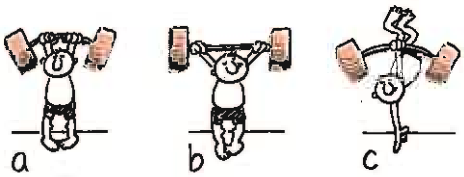


Fig. B

5. Why do electric power lines sometimes break in winter when a small weight of ice forms on them?

6. Why cannot the strong man in Figure C pull hard enough to make the chain straight?

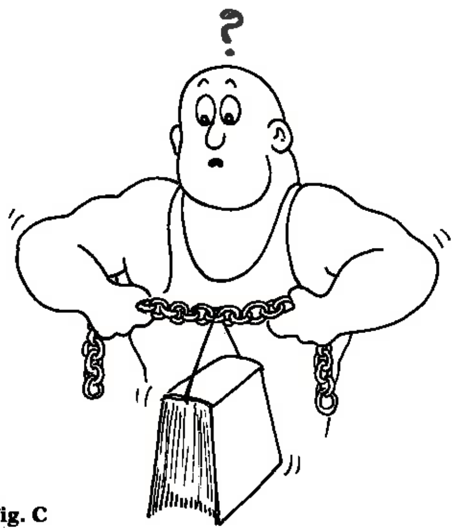


Fig. C

7. Why are the main supporting cables of suspension bridges designed to sag the way they do (Figure D)?

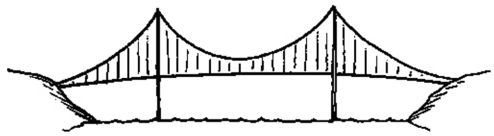


Fig. D

8. The boy on the tower (Figure E) throws a ball 20 m downrange, as shown. What is his pitching speed?

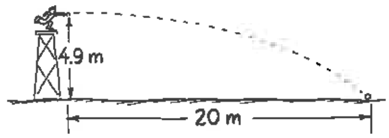


Fig. E

9. Why does a ball rolling down an incline undergo more acceleration the steeper the incline?

10. Why is less force required to push a barrel up a sloping ramp (Figure F) than to lift it vertically?

Fig. F